NEHRU COLLEGE OF ENGINEERING AND RESEARCH CENTRE
(NAAC Accredited)
(Approved by AICTE, Affiliated to APJ Abdul Kalam Technological University, Kerala)

DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

COURSE MATERIALS

EE469 ELECTRIC AND HYBRID VEHICLES

VISION OF THE INSTITUTION

To mould true citizens who are millennium leaders and catalysts of change through excellence in education.

MISSION OF THE INSTITUTION

NCERC is committed to transform itself into a center of excellence in Learning and Research in Engineering and Frontier Technology and to impart quality education to mould technically competent citizens with moral integrity, social commitment and ethical values.

We intend to facilitate our students to assimilate the latest technological know-how and to imbibe discipline, culture and spiritually, and to mould them in to technological giants, dedicated research scientists and intellectual leaders of the country who can spread the beams of light and happiness among the poor and the underprivileged.

COURSE OBJECTIVES

Course Name: EE469 ELECTRTIC AND HYBRID VEHICLES

YEAR of Study: FOURTH YEAR

C469.1	**Understand about evolution of Conventional and Hybrid Electric Vehicles.**
C469.2	Understand about the hybrid electric drive trains and electric drive trains.
C469.3	Gain knowledge in Electric Propulsion unit.
C469.4	Gain knowledge in Energy storage.
C469.5	Understand the sizing of drive train system
C469.6	Give knowledge about energy management strategies and communications.

PROGRAM OUTCOMES (POs)

Engineering Graduates will be able to:

1. **Engineering knowledge**: Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.
2. **Problem analysis**: Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.
3. **Design/development of solutions**: Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.
4. **Conduct investigations of complex problems**: Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.
5. **Modern tool usage**: Create, select, and apply appropriate techniques,

resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.

6. **The engineer and society**: Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.
7. **Environment and sustainability**: Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.
8. **Ethics**: Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.
9. **Individual and team work**: Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.
10. **Communication**: Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clearinstructions.
11. **Project management and finance**: Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinaryenvironments.
12. **Life-long learning**: Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technologicalchange.

CO-PO matrices of courses

CO	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
C469.1	3	2	3	3	3	3	3	2	-	-	-	3
C469.2	3	2	3	3	3	3	3	2	-	-	-	3

C469.3	3	3	3	2	3	2	2	1	-	-	-	3
C469.4	3	3	3	2	3	2	2	1	-	-	-	3
C469.5	3	3	3	2	3	2	2	1	-	-	-	3
C469.6	3	3	3	2	3	2	2	1	-	-	-	3
C469	3	2.66	3	2.33	3	2.33	2.33	1.33	-	-	-	3

CO AND PSO MAPING

PSO

1. Apply Science, Engineering, Mathematics through differential and Integral Calculus, Complex Variables to solve Electrical Engineering Problems.

2. Demonstrate proficiency in the use of software and hardware to be required to practice electrical engineering profession.

3. Able to apply the knowledge of Ethical and Management principles required to work in

CO/PSO	PSO1	PSO2	PSO3
C469.1	3	3	1
C469.2	3	3	1
C469.3	3	3	1
C469.4	3	3	1
C469.5	3	3	1
C469.6	3	3	1
C469	3	3	1

Module 1

Introduction to Hybrid Electric Vehicles

Introduction:

What is a hybrid? A hybrid vehicle combines any two power (energy) sources. Possible combinations include diesel/electric, gasoline/fly wheel, and fuel cell (FC)/battery. Typically, one energy source is storage, and the other is conversion of a fuel to energy. The combination of two power sources may support two separate propulsion systems. Thus to be a True hybrid, the vehicle must have at least two modes of propulsion.

For example, a truck that uses a diesel to drive a generator, which in turn drives several electrical motors for all-wheel drive, is *not a hybrid*. But if the truck has electrical energy storage to provide a second mode, which is electrical assists, then it is a hybrid Vehicle.

These two power sources may be paired in series, meaning that the gas engine charges the batteries of an electric motor that powers the car, or in parallel, with both mechanisms driving the car directly.

Hybrid electric vehicle (HEV)

Consistent with the definition of hybrid above, the hybrid electric vehicle combines a gasoline engine with an electric motor. An alternate arrangement is a diesel engine and an electric motor (figure 1).

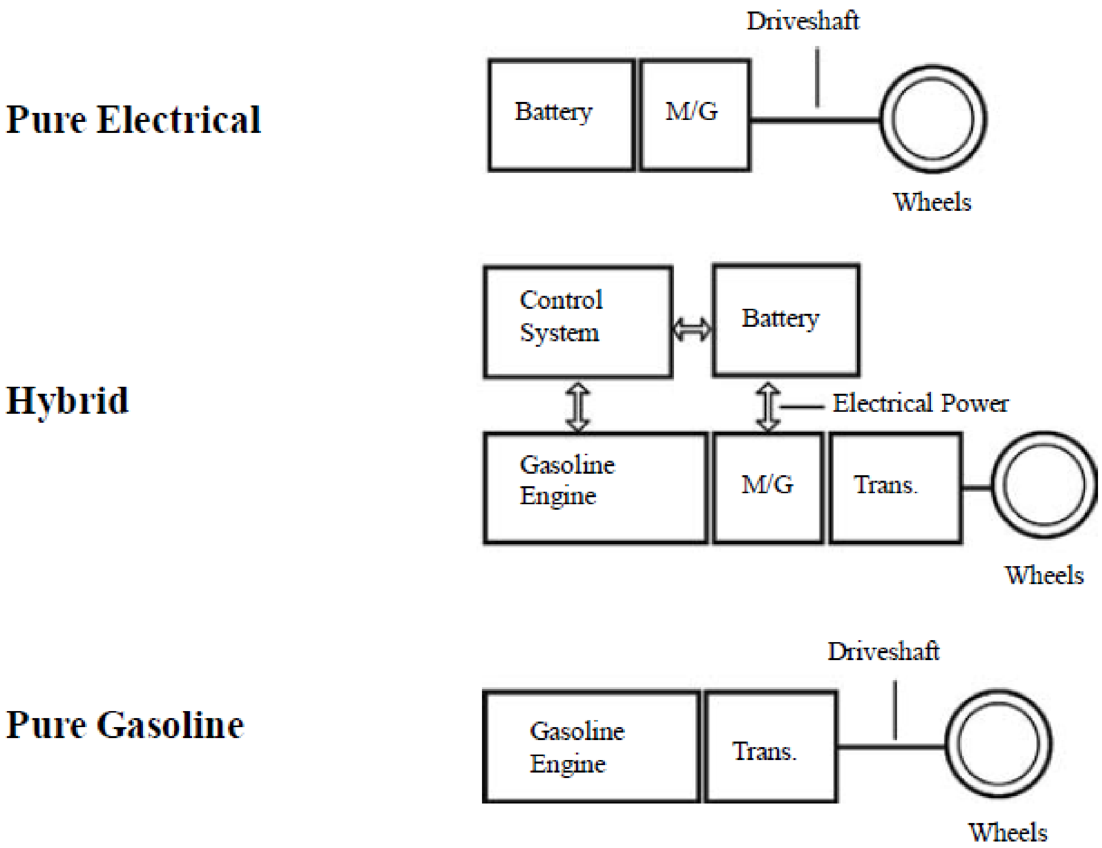

Figure 1: Components of a hybrid Vehicle that combines a pure gasoline with a pure EV. [1]

As shown in **Figure 1**, a HEV is formed by merging components from a pure electrical vehicle and a pure gasoline vehicle. The Electric Vehicle (EV) has an M/G which allows regenerative braking for an EV; the M/G installed in the HEV enables regenerative braking. For the HEV, the M/G is tucked directly behind the engine. In Honda hybrids, the M/G is connected directly to the engine. The transmission appears next in line. This arrangement has two torque producers; the M/G in motor mode, M-mode, and the gasoline engine. The battery and M/G are connected electrically.

HEVs are a combination of electrical and mechanical components. Three main sources of electricity for hybrids are batteries, FCs, and capacitors. Each device has a low cell voltage, and, hence, requires many cells in series to obtain the voltage demanded by an HEV. Difference in the source of Energy can be explained as:

a. The FC provides high energy but low power.

b. The battery supplies both modest power and energy.

c. The capacitor supplies very large power but low energy.

The components of an electrochemical cell include anode, cathode, and electrolyte (shown in fig2). The current flow both internal and external to the cell is used to describe the current loop.

e−

External

circuit

Anode Cathode

Electrolyte

Electrodes

Figure 2: An electrode, a circuit for a cell which is converting chemical energy to electrical energy. The motion of negative charges is clockwise and forms a closed loop through external wires and load and the electrolyte in the cell. [1]

A critical issue for both battery life and safety is the precision control of the Charge/Discharge cycle. Overcharging can be traced as a cause of fire and failure. Applications impose two boundaries or limitations on batteries. The first limit, which is dictated by battery life, is the minimum allowed State of Charge. As a result, not all the installed battery energy can be used. The battery feeds energy to other electrical equipment, which is usually the inverter. This equipment can use a broad range of input voltage, but cannot accept a low voltage. The second limit is the minimum voltage allowed from the battery.

Historical development (root) of Automobiles

In 1900, steam technology was advanced. The advantages of ***steam-powered cars*** included high performance in terms of power and speed. However, the disadvantages of steam-powered cars included poor fuel economy and the need to “fire up the boiler” before driving. Feed water was a necessary input for steam engine, therefore could not tolerate the loss of fresh water. Later, Steam condensers were applied to the steam car to solve the feed water problem. However, by that time Gasoline cars had won the marketing battle.

Gasoline cars of 1900 were noisy, dirty, smelly, cantankerous, and unreliable. In comparison, electric cars were comfortable, quiet, clean, and fashionable. Ease of control was also a desirable feature. Lead acid batteries were used in 1900 and are still used in modern cars. Hence lead acid atteries have a long history (since 1881) of use as a viable energy storage device. Golden age of ***Electrical vehicle*** marked from 1890 to 1924 with peak production of electric vehicles in 1912. However, the range was limited by energy storage in the battery. After every trip, the battery required recharging. At the 1924 automobile show, no electric cars were on display. This announced the end of the Golden Age of electric-powered cars.

The range of a ***gasoline car*** was far superior to that of either a steam or an electric car and dominated the automobile market from 1924 to 1960. The gasoline car had one dominant feature; it used gasoline as a fuel. The modern period starts with the oil embargoes and the gasoline shortages during the 1970s which created long lines at gas stations. Engineers recognized that the good features of the gasoline engine could be combined with those of the electric motor to produce a superior car. A marriage of the two yields the hybrid automobile.

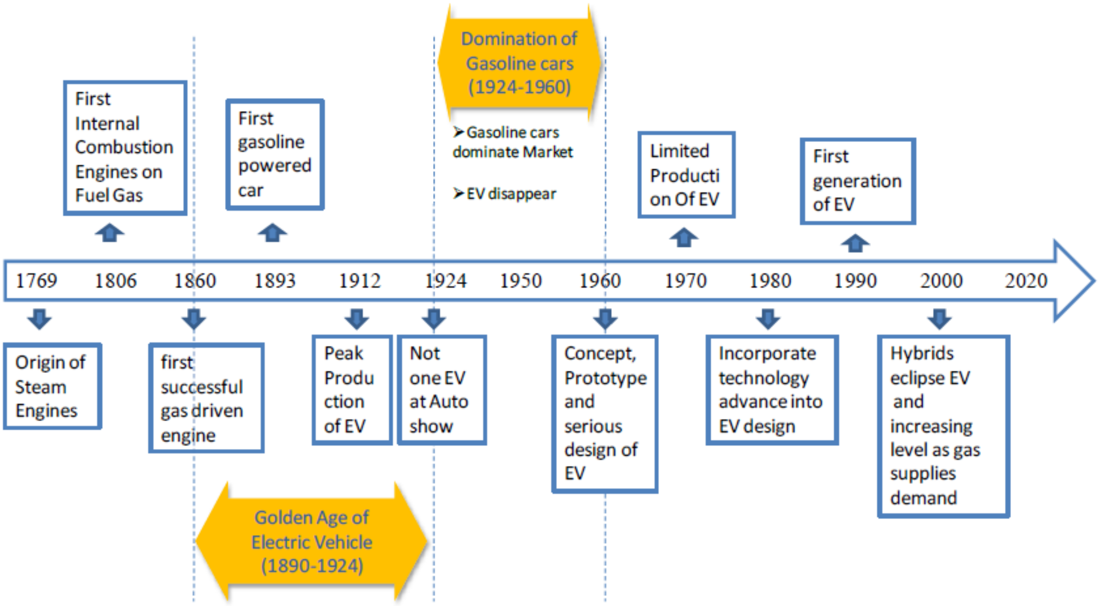

Figure 3: Historical development of automobile and development of interest and activity in the EV from 1890 to present day. Electric Vehicle merged into hybrid electric vehicle. [1]

1769

The ***first steam-powered vehicle*** was designed by Nicolas-Joseph Cugnot and constructed by M. Brezin that could attain speeds of up to 6 km/hour. These early steam-powered vehicles were so heavy that they were only practical on a perfectly flat surface as strong as iron.

1807

The next step towards the development of the car was the invention of the internal combustion engine. Francois Isaac de Rivaz designed the ***first internal combustion engine*** in, using a mixture of hydrogen and oxygen to generate energy.

1825

British inventor Goldsworthy Gurney built a steam car that successfully completed an 85 mile round-trip journey in ten hours time.

1839

Robert Anderson of Aberdeen, Scotland built the ***first electric vehicle***.

1860

In, Jean Joseph Etienne Lenoir, a Frenchman, built the ***first successful two-stroke gas driven engine***.

1886

Historical records indicate that ***an electric-powered taxicab***, using a battery with 28 cells and a small electric motor, was introduced in England.

1888

Immisch & Company built a four-passenger carriage, powered by a one-horsepower motor and 24-cell battery, for the Sultan of the Ottoman Empire. In the same year, Magnus Volk in Brighton, England made a three-wheeled electric car. ***1890 – 1910 (Period of significant improvements in battery technology)***

Invention Of hybrid vehicle

1890

Jacob Lohner, a coach builder in Vienna, Austria, foresaw the need for an electric vehicle that would be less noisy than the new gas-powered cars. He commissioned a design for an electric vehicle from Austro-Hungarian engineer Ferdinand Porsche, who had recently graduated from the Vienna Technical College. Porsche's first version of the electric car used a pair of electric motors mounted in the front wheel hubs of a conventional car. The car could travel up to 38 miles. To extend the vehicle's range, Porsche added a gasoline engine that could recharge the batteries, thus giving birth to the first hybrid, the ***Lohner-Porsche Elektromobil***.

Early Hybrid Vehicles

1900

Porsche showed his hybrid car at the Paris Exposition of 1900. A gasoline engine was used to power a generator which, in turn, drove a small series of motors. The electric engine was used to give the car a little bit of extra power. This method of ***series hybrid engine*** is still in use today, although obviously with further scope of performance improvement and greater fuel savings.

1915

Woods Motor Vehicle manufacturers created the Dual Power hybrid vehicle, second hybrid car in market. Rather than combining the two power sources to give a single output of power, the Dual Power used an electric battery motor to power the engine at low speeds (below 25km/h) and used the gasoline engine to carry the vehicle from these low speeds up to its 55km/h maximum speed. While Porsche had invented the series hybrid, Woods invented the parallel hybrid.

1918

The Woods Dual Power was the ***first hybrid to go into mass production***. In all, some 600 models were built by. However, the evolution of the internal combustion engine left electric power a marginal technology

1960

Victor Wouk worked in helping create numerous hybrid designs earned him the nickname of the "Godfather of the Hybrid". In 1976 he even converted a Buick Skylark from gasoline to hybrid.

1978

Modern hybrid cars rely on the regenerative braking system. When a standard combustion engine car brakes, a lot of power is lost because it dissipates into the atmosphere as heat. Regenerative braking means that the electric motor is used for slowing the car and it essentially collects this power and uses it to help recharge the electric batteries within the car. This development alone is believed to have progressed hybrid vehicle manufacture significantly. The Regenerative Braking System, was first designed and developed in 1978 by David Arthurs. Using standard car components he converted an Opel GT to offer 75 miles to the gallon and many home conversions are done using the plans for this system that are still widely available on the Internet.

Modern Period of Hybrid History

The history of hybrid cars is much longer and more involved than many first imagine. It is, however, in the last ten years or so that we, as consumers, have begun to pay more attention to the hybrid vehicle as a viable alternative to ICE driven cars. Whether looking for a way to save money on spiraling gas costs or in an attempt to help reduce the negative effects on the environment we are buying hybrid cars much more frequently.

1990s

Automakers took a renewed interest in the hybrid, seeking a solution to dwindling energy supplies and environmental concerns and created modern history of hybrid car

1993

In USA, Bill Clinton's administration recognized the urgency for the mass production of cars powered by means other than gasoline. Numerous government agencies, as well as Chrysler, Ford, GM, and USCAR combined forces in the PNGV (Partnership for a New Generation of Vehicles), to create cars using alternative power sources, including the development and improvement of hybrid electric vehicles.

1997

The Audi Duo was the first European hybrid car put into mass production and hybrid production and consumer take up has continued to go from strength to strength over the decades.

2000

Toyota Prius and Honda Insight became the first mass market hybrids to go on sale in the United States, with dozens of models following in the next decade. The Honda Insight and Toyota Prius were two of the first mainstream Hybrid Electric Vehicles and both models remain a popular line.

2005

A hybrid Ford Escape, the SUV, was released in 2005. Toyota and Ford essentially swapped patents with one another, Ford gaining a number of Toyota patents relating to hybrid technology and Toyota, in return, gaining access to Diesel engine patents from Ford.

Present of Hybrid Electric vehicle

Toyota is the most prominent of all manufacturers when it comes to hybrid cars. As well as the specialist hybrid range they have produced hybrid versions of many of their existing model lines, including several Lexus (now owned and manufactured by Toyota) vehicles. They have also stated that it is their intention to release a hybrid version of every single model they release in the coming decade. As well as cars and SUVs, there are a select number of hybrid motorcycles, pickups, vans, and other road going vehicles available to the consumer and the list is continually increasing.

Future of Hybrid electrical vehicle

Since petroleum is limited and will someday run out of supply. In the arbitrary year 2037, an estimated one billion petroleum-fueled vehicles will be on the world's roads. gasoline will become prohibitively expensive. The world need to have solutions for the "***400 million otherwise useless cars"***. So year 2037 "gasoline runs out year" means, petroleum will no longer be used for personal mobility. A market may develop for solar-powered

EVs of the size of a scooter or golf cart. Since hybrid technology applies to heavy vehicles, hybrid buses and hybrid trains will be more significant.

Economic and Environmental Impact of Electric Hybrid Vehicle

As modern culture and technology continue to develop, the growing presence of global warming and irreversible climate change draws increasing amounts of concern from the world's population. It has only been recently, when modern society has actually taken notice of these changes and decided that something needs to change if the global warming process is to be stopped.

Countries around the world are working to drastically reduce CO_2 emissions as well as other harmful environmental pollutants. Amongst the most notable producers of these pollutants are automobiles, which are almost exclusively powered by internal combustion engines and spew out unhealthy emissions.

According to various reports, cars and trucks are responsible for almost 25% of CO_2 emission and other major transportation methods account for another 12%. With immense quantities of cars on the road today, pure combustion engines are quickly becoming a target of global warming blame. One potential alternative to the world's dependence on standard combustion engine vehicles are hybrid cars. Cost-effectiveness is also an important factor contributing to the development of an environment friendly transportation sector.

Hybrid Vehicle

A hybrid vehicle combines any type of two power (energy) sources. Possible combinations include diesel/electric, gasoline/fly wheel, and fuel cell (FC)/battery. Typically, one energy source is storage, and the other is conversion of a fuel to energy. In the majority of modern hybrids, cars are powered by a combination of traditional gasoline power and the addition of an electric motor.

However, hybrid still use the petroleum based engine while driving so they are not completely clean, just cleaner than petroleum only cars. This enables hybrid cars to have the potential to segue into new technologies that rely strictly on alternate fuel sources.

The design of such vehicles requires, among other developments, improvements in power train systems, fuel processing, and power conversion technologies. Opportunities for utilizing various fuels for vehicle propulsion, with an emphasis on synthetic fuels (e.g., hydrogen, biodiesel, bioethanol, dimethylether, ammonia, etc.) as well as electricity via electrical batteries, have been analyzed over the last decade.

Economical Analysis

A number of key economic parameters that characterize vehicles were:

A. Vehicle price,

B. Fuel cost, and

C. Driving range.

This case neglected maintenance costs; however, for the hybrid and electric vehicles, the cost of battery replacement during the lifetime was accounted for. The driving range determines the frequency (number and separation distance) of fueling stations for each vehicle type. The total fuel cost and the total number of kilometers driven were related to the vehicle life (see Table 1).

Table1: Technical and economical values for selected vehicle types

Vehicle type	Fuel Type	Initial Price (USk$)	Specific fuel Price (US$/100 km)	Driving Range (Km)	Price of battery Changes During Vehicle Life cycle (USk$)
Conventional *(Toyota Corolla)*	Gasoline	15.3	2.94	540	1 x 0.1
Hybrid *(Toyota Prius)*	Gasoline	20	1.71	930	1 x 1.02
Electric *(Toyota RAV4EV)*	Electricity	42	0.901	164	2 x 15.4
Fuel cell *(Honda FCX)*	Hydrogen	100	1.69	355	1 x 0.1
H2-ICE *(Ford Focus H2-ICE)*	Hydrogen	60	8.4	300	1 x 0.1
NH3–H2-ICE *(Ford Focus H2-ICE and ammonia Adaptive)*	Ammonia	40	6.4	430	1 x 0.1

For the Honda FCX the listed initial price for a prototype leased in 2002 was USk$2,000, which is estimated to drop below USk$100 in regular production. Currently, a Honda FCX can be leased for 3 years with a total price of USk$21.6. In order to render the comparative study reasonable, the initial price of the hydrogen fuel cell vehicle is assumed here to be USk$100. For electric vehicle, the specific cost was estimated to be US$569/kWh with nickel metal hydride (NiMeH) batteries which are typically used in hybrid and electric cars.

Historical prices of typical fuels were used to calculate annual average price.

Environmental Analysis

Analysis for the first five options was based on published data from manufacturers. The results for the sixth case, i.e. the ammonia-fueled vehicle, were calculated from data published by Ford on the performance of its hydrogen-fueled Ford Focus vehicle. Two environmental impact elements were accounted for in the:

a) Air pollution (AP) and

b) Greenhouse gas (GHG) emissions.

The main GHGs were CO_2, CH_4, N_2O, and SF_6 (sulfur hexafluoride), which have GHG impact weighting coefficients relative to CO_2 of 1, 21, 310, and 24,900, respectively.

For AP, the airborne pollutants CO, NO_x, SO_x, and VOCs are assigned the following weighting coefficients: 0.017, 1, 1.3, and 0.64, respectively.

The vehicle production stage contributes to the total life cycle environmental impact through the pollution associated with

a) The extraction and processing of material resources,

b) Manufacturing and

c) The vehicle disposal stage.

Additional sources of GHG and AP emissions were associated with the fuel production and utilization stages. The environmental impacts of these stages have been evaluated in numerous life cycle assessments of fuel cycles.

Regarding electricity production for the electric car case, three case scenarios were considered here:

1. when electricity is produced from renewable energy sources and nuclear energy;
2. when 50% of the electricity is produced from renewable energy sources and 50% from natural gas at an efficiency of 40%;
3. when electricity is produced from natural gas at an efficiency of 40%.

AP emissions were calculated assuming that GHG emissions for plant manufacturing correspond entirely to natural gas combustion. GHG and AP emissions embedded in manufacturing a natural gas power generation plant were negligible compared to the direct emissions during its utilization. Taking those factors into account, GHG and AP emissions for the three scenarios of electricity generation were presented in Table 2.

Table2: GHG and air pollution emissions per MJ of electricity produced

Electricity-generation scenario	Description of Electricity generation Scenario	GHG emission (g)	AP emission (g)
1	Electricity produced = 100% (Renewable Energy + Nuclear Energy)	5.11	0.195
2	Electricity produced = (50% Renewable Energy + 50% Natural gas)	77.5	0.296
3	Electricity produced = 100% Natural Gas	149.9	0.573

Hydrogen charging of fuel tanks on vehicles requires compression. Therefore, presented case considered the energy for hydrogen compression to be provided by electricity.

Table 3: GHG and air pollution emissions per MJ fuel of Hydrogen from natural gas produced

Fuel	GHG emissions, g	AP emissions, g
Hydrogen from natural gas		
Scenario 1	78.5	0.0994
Scenario 2	82.1	0.113
Scenario 3	85.7	0.127

GHG and AP emissions were reported for hydrogen vehicles for the three electricity-generation scenarios considered (see table 3), accounting for the environmental effects of hydrogen compression

Table 4. Environmental impact associated with vehicle Overall Life cycle and Fuel Utilization State

	Fuel utilization stage		Overall life cycle	
Vehicle type	GHG emissions (kg/100 km)	AP emissions (kg/100 km)	GHG emissions (kg/100 km)	AP emissions (kg/100 km)
Conventional	19.9	0.0564	21.4	0.06
Hybrid	11.6	0.0328	13.3	0.037
Electric-S1	0.343	0.00131	2.31	0.00756
Electric-S2	5.21	0.0199	7.18	0.0262
Electric-S3	10.1	0.0385	12	0.0448
Fuel Cell -S1	10.2	0.0129	14.2	0.0306
Fuel Cell -S2	10.6	0.0147	14.7	0.0324
Fuel Cell -S3	11.1	0.0165	15.2	0.0342
H2-ICE	10	0.014	11.5	0.018
NH3–H2-ICE	0	0.014	1.4	0.017

The environmental impact of the fuel utilization stage, as well as the overall life cycle is presented in Table 4. The H2-ICE vehicle results were based on the assumption that the only GHG emissions during the utilization stage were associated with the compression work, needed to fill the fuel tank of the vehicle. The GHG effect of water vapor emissions was neglected in this analysis due its little value,. For the ammonia fuel vehicle, a very small amount of pump work was needed therefore, ammonia fuel was considered to emit no GHGs during fuel utilization.

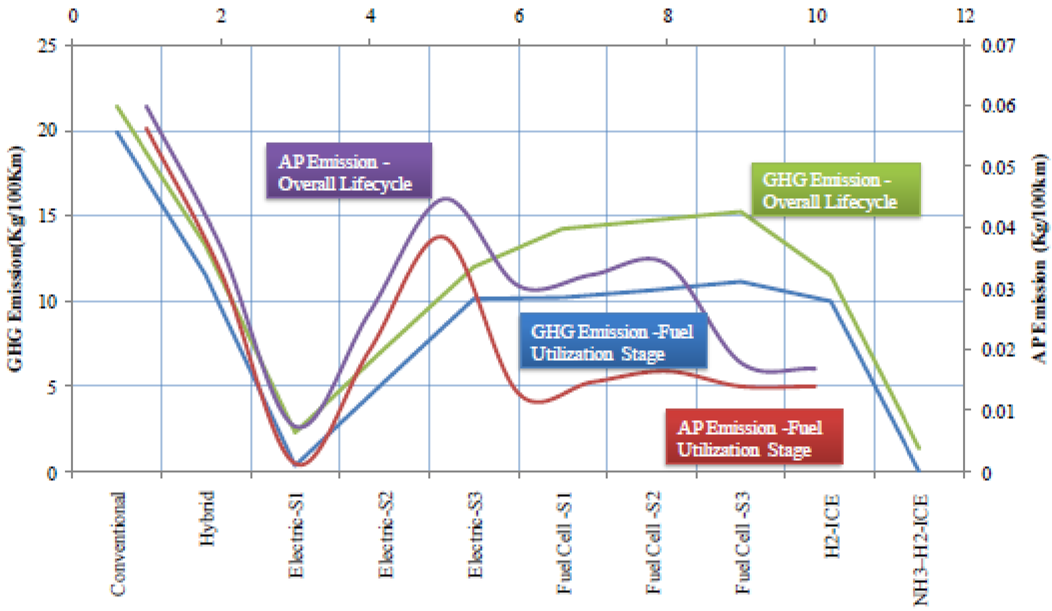

Figure1: Environmental indicators for six vehicle types [1]

Results of technical–economical–environmental Analysis:

In present situation this case study provides a general approach for assessing the combined technical–economical–environmental benefits of transportation options.

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Module 2: Dynamics of Electric and Hybrid vehicles

Lecture 3: Motion and dynamic equations for vehicles

Motion and dynamic equations for vehicles

Introduction

The fundamentals of vehicle design involve the basic principles of physics, specially the Newton's second law of motion. According to Newton's second law the acceleration of an object is proportional to the net force exerted on it. Hence, an object accelerates when the net force acting on it is not zero. In a vehicle several forces act on it and the net or resultant force governs the motion according to the Newton's second law. The propulsion unit of the vehicle delivers the force necessary to move the vehicle forward. This force of the propulsion unit helps the vehicle to overcome the resisting forces due to gravity, air and tire resistance. The acceleration of the vehicle depends on:

δ. the power delivered by the propulsion unit

ε. the road conditions

ϕ. the aerodynamics of the vehicle

γ. the composite mass of the vehicle

In this lecture the mathematical framework required for the analysis of vehicle mechanics based on Newton’s second law of motion is presented. The following topics are covered in this lecture:

η. General description of vehicle movement

ι. Vehicle resistance

φ. Dynamic equation

κ. Tire Ground Adhesion and maximum tractive effort

General description of vehicle movement

The vehicle motion can be completely determined by analysing the forces acting on it in the direction of motion. The forces acting on a vehicle, moving up a grade, are shown in **Figure 1**. The tractive force (**F_t**) in the contact area between the tires of the driven wheels and the road surface propels the vehicle forward. The tractive force (**F_t**) is produced by the power plant and transferred to the driving wheels via the transmission and the final drive. When the vehicle moves, it encounters a resistive force that tries to retard its motion. The resistive forces are

- Rolling resistance
- Aerodynamic drag
- Uphill resistance

V

F_w

h_w

Mgsin(α)

T_{rf}

F_a

h_g

T_{rr}

Mgcos(α)

W_f

Mg

L_a

L_b

α

W_r

L

Figure 1: Forces acting on a vehicle going uphill [1]

Using the Newton's second law of motion, the vehicle acceleration can be expressed as

$$\frac{dV}{dt} = \frac{\sum F_t - \sum F_{resistance}}{\delta M} \qquad (1)$$

where

V = vehicle speed

$\sum F_t$ = total tractive effort [Nm]

$\sum F_{resistance}$ = total resistance [Nm]

M = total mass of the vehicle [kg]

δ = mass factor for converting the rotational inertias of rotating components into translational mass

Rolling resistance

The rolling resistance of tires on hard surfaces is due to hysteresis in the tire material. In **Figure 2** a tire at standstill is shown. On this tyre a force (**P**), is acting at its centre. The pressure in the contact area between the tire and the ground is distributed symmetrically to the centre line and the resulting reaction force (**P_z**) is aligned along **P**.

P

r

z

P_z

Figure 2: Pressure distribution in contact area [1]

The deformation, **z**, versus the load **P**, in the loading and unloading process is shown in **Figure 3**. From this figure it can be seen that, due to the hysteresis, the force (**P**) for the same deformation (**z**) of the tire material at loading is greater than at during unloading. Hence, the hysteresis causes an asymmetric distribution of the ground reaction forces.

Force, P

P_1

P_2

Deformation, z

Figure 3: Force acting on a tyre vs. deformation in loading and unloading [1]

The scenario of a rolling tire is shown in **Figure 4**. When the tire rolls, the leading half of the contact area is loading and the trailing half is unloading. Thus, the pressure on the leading half is greater than the pressure on the trailing half (**Figure 4a**). This phenomenon results in the ground reaction force shifting forward. The shift in the ground reaction force creates a moment that opposes rolling of the wheels. On soft surfaces, the rolling resistance is mainly caused by deformation of the ground surface, (**Figure 4b**). In this case the ground reaction force almost completely shifts to the leading half.

P

r

z

a

P z

Figure 4a: Force acting on a tyre vs. deformation in loading and unloading on a hard surface [1]

The moment produced by forward shift of the resultant ground reaction force is called rolling resistance moment (**Figure 4a**) and can expressed as

$$T_r = Pa = Mga$$

where

T_r = rolling resistance [Nm]

P = Normal load acting on

the centre of the rolling wheel [N]

M = mass of the vehicle [kg]

(2)

g = acceleration constant [m/s^2]

a = deformation of the tyre [m]

P

r

P_x z

P_z

Figure 4a: Force acting on a tyre vs. deformation in loading and unloading on a soft surface [1]

To keeps the wheel rolling, a force $\mathbf{F_r}$, acting on the centre of the wheel is required to balance this rolling resistant moment. This force is expressed as

$$F_r = \frac{T_r}{r_{dyn}} = \frac{Pa}{r_{dyn}} = Pf_r \quad (3)$$

where

T_r = rolling resistance [Nm]

P = Normal load acting on the centre of the rolling wheel [N]

r_{dyn} = dynamic radius of the tyre [m]

f_r = rolling resistance coefficient

The rolling resistance moment can be equivalently replaced by horizontal force acting on the wheel centre in the direction opposite to the movement of the wheel. This equivalent force is called the rolling resistance and its magnitude is given by

$$F_r = Pf_r$$

where (4)

P = Normal load acting on the centre of the rolling wheel
$[N]$ f_r = rolling resistance coefficient

When a vehicle is moving up a gradient, the normal force (**P**), in **equation 4**, is replaced by the component that is perpendicular to the road surface. Hence, **equation 4** is rewritten as

$$F_r = Pf_r\cos(\alpha) = Mgf_r\cos(\alpha)$$

where

(5)

P = Normal load acting on the centre of the rolling
wheel $[N]$ f_r = rolling resistance coefficient

α = road angle [*radians*]

.

The rolling resistance coefficient, $\mathbf{f_r}$, is a function of:

Δ. tire material

E. tire structure

Φ. tire temperature

Γ. tire inflation pressure

H. tread geometry

I. road roughness

ϑ. road material

K. presence of absence of liquids on the road

The typical values of the rolling resistance coefficient ($\mathbf{f_r}$) are given in **Table 1**.

Table 1: Reference values for the rolling resistance coefficient (f_r)

Conditions	**Rolling resistance coefficient (f_r)**
Car tire on smooth tarmac road	0.01
Car tire on concrete road	0.011
Car tire on a rolled gravel road	0.02
Tar macadam road	0.025
Unpaved road	0.05
Bad earth tracks	0.16

Loose sand	0.15-0.3
Truck tire on concrete or asphalt road	0.006-0.01
Wheel on iron rail	0.001-0.002

The values given in table 1 do not take into account the variation of $\mathbf{f_r}$ with speed. Based on experimental results, many empirical formulas have been proposed for calculating the rolling resistance on a hard surface. For example, the rolling resistance coefficient of a passenger car on a concrete road may be calculated as:

$$f_r = f_0 + f_s \left(\frac{V}{100} \right)^{2.5}$$

where (6)

$v)$= vehicle speed $[km / h]$

In vehicle performance calculation, it is sufficient to consider the rolling resistance coefficient as a linear function of speed. For most common range of inflation pressure, the following equation can be used for a passenger car on a concrete road

$$f_r = 0.01\left(1 + \frac{V}{160}\right) \qquad (7)$$

where

$v)$= vehicle speed $[km / h]$

The **equation 7** can predict the values of $\mathbf{f_r}$ with acceptable accuracy for speed up to 128km/h. **Aerodynamic drag**

A vehicle traveling at a particular speed in air encounters a force resisting its motion. This force is known as aerodynamic drag. The main causes of aerodynamic drag are:

4. shape drag
5. skin effect

The shape drag is due to the shape of the vehicle. The forward motion of the vehicle pushes the air in front of it. However, the air cannot instantaneously move out of the way and its pressure is thus increased. This results in high air pressure in the front of the vehicle. The air behind the vehicle cannot instantaneously fill the space left by the forward motion of the vehicle. This creates a zone of low air pressure. Hence, the motion of the vehicle creates two zones of pressure. The high pressure zone in the front of the vehicle opposes its movement by pushing. On the other hand, the low pressure zone developed at the rear of the vehicle opposes its motion by pulling it backwards.

The air close to the skin of the vehicle moves almost at the speed of the vehicle while the air away from the vehicle remains still. Between these two layers (the air layer moving at the vehicle speed and the static layer) the molecules move at a wide range of speeds. The difference in speed between two air molecules produces friction. This friction results in the second component of aerodynamic drag and it is known as skin effect.

The aerodynamic drag is expressed as

$$F_w = \frac{1}{2}\rho A_f C_D V^2 \qquad (8)$$

where

ρ = density of air [kg/m^3]

A_f = vehicle frontal area [m^2]

V = vehicle speed [m/s] C_D = drag coefficient

.

The aerodynamic drag coefficients and the frontal area for different vehicle types are given in **Table 2**.

Table 2: Reference values for drag coefficient (C_D) and

the frontal area (A_f in m^2) for some vehicle types

Vehicle	C_D	A_f
Motorcycle with rider	0.5-0.7	0.7-0.9
Open convertible	0.5-0.7	1.7-2.0
Limousine	0.22-0.4	1.7-2.3
Coach	0.4-0.8	6-10
Truck without trailer	0.45-0.8	6.0-10.0
Truck with trailer	0.55-1.0	6.0-10.0
Articulated vehicle	0.5-0.9	6.0-10.0

Grading resistance

When a vehicle goes up or down a slope, its weight produces a component of force that is always directed downwards, **Figure 5**. This force component opposes the forward motion, i.e.

the grade climbing. When the vehicle goes down the grade, this force component aids the vehicle motion. The grading resistance can be expressed as

$$F_g = Mg\sin(\alpha) \qquad (9)$$

where

M = mass of vehicle [kg]

g = acceleration constant [m/s^2]
α = road angle [$radians$]

In order to simplify the calculation, the road angle α, is usually replaced by the grade value, when the road angle is small. The grade value is defined as (**Figure 5**)

$$i = \frac{H}{L} = \tan(\alpha) \approx \sin(\alpha) \qquad (10)$$

.

In some literature, the tire rolling resistance and the grading resistance taken together and is called **road resistance.** The road resistance is expressed as

$$F_{rd} = F_f + F_g = Mg\,(f_r \cos(\alpha) + \sin(\alpha)) \qquad (11)$$

where

M = mass of vehicle $[kg]$

g = acceleration constant$[m/s^2]$

f_r = rolling resistance coefficient

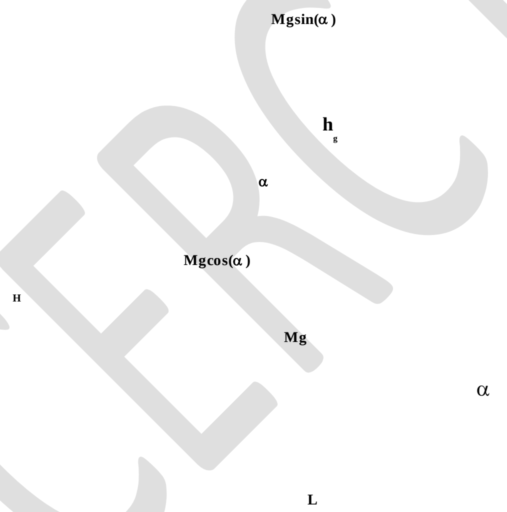

Figure 5: Vehicle going up a grade [1]

Acceleration resistance

In addition to the driving resistance occurring in steady state motion, inertial forces also occur during acceleration and braking. The total mass of the vehicle and the inertial mass of those rotating parts of the drive accelerated or braked are the factors influencing the resistance to acceleration:

$$F = \left(M + \frac{\sum J_{rot}}{r} \right) \frac{dV}{dt}$$

$$F_a = \left(\quad \quad ^2 \quad \right)_{dyn}$$

where

(12)

M = mass of vehicle $[kg]$

J_{rot} = intertia of rotational components $[kg \times m^2]$

V = speed of the vehicle $[km/h]$

r_{dyn} = *dynamic radius of the tyre* $[m]$

The rotational component is a function of the gear ratio. The moment of inertia of the rotating drive elements of engine, clutch, gearbox, drive shaft, etc., including all the road wheels are reduced to the driving axle. The acceleration resistance can be expressed as

$$F_a = \lambda M \frac{dV}{dt} \qquad (13)$$

where

λ = rotational inertia constant

M = mass of the vehicle $[kg]$ V = speed of the vehicle $[m/s]$

Total driving resistance

The traction force (**F_t**) required at the drive wheels is made up of the driving resistance forces and is defined as

$$F_{resistance} = F_r + F_w + F_g + F_a \quad (14)$$

Substituting the values of all the forces in equation 14, gives

$$F_{resistance} = Mgf_r \cos(\alpha) + \frac{1}{2}\rho A_f C_D V^2 + Mg\sin(\alpha) + \lambda M \frac{dV}{dt} \quad (15)$$

The equation 15 may be used to calculate the power required (**P_{req}**):

$$P_{req} = F_{resistance} V \quad (16)$$

Dynamic equation

In the longitudinal direction, the major external forces acting on a two axle vehicle (**Figure 1**)

include:

- the rolling resistance of the front and rear tires (**F_{rf}** and **F_{rr}**), which are represented by rolling resistance moment, **T_{rf}** and **T_{rr}**
- the aerodynamic drag (**F_w**)
- grade climbing resistance (**F_g**)
- acceleration resistance (**F_a**)

The dynamic equation of vehicle motion along the longitudinal direction is given by

$$M\frac{dV}{dt} = \left(F_{tf} + F_{tr}\right) - \left(F_{rf} + F_{rr} + F_w + F_g + F_a\right) \quad (17)$$

The first term on the right side is the total tractive effort and the second term is the total tractive resistance. To determine the maximum tractive effort, that the tire ground contact can support, the normal loads on the front and rear axles have to be determined. By summing the moments of all the forces about point **R** (centre of the tire-ground area), the normal load on the front axle $\mathbf{W_f}$ can be determined as

$$W_f = \frac{MgL_b\cos(\alpha) - \left(T_{rf} + T_{rr} + F_w h_w + Mgh_g\sin(\alpha) + Mh_g\frac{dV}{dt}\right)}{L} \qquad (18)$$

Similarly, the normal load acting on the rear axle can be expressed as

$$W_r = \frac{MgL_a\cos(\alpha) - \left(T_{rf} + T_{rr} + F_w h_w + Mgh_g\sin(\alpha) + Mh_g\frac{dV}{dt}\right)}{L} \qquad (19)$$

In case of passenger cars, the height of the centre of application of aerodynamic resistance (h_W) is assumed to be near the height of centre of gravity of the vehicle (h_g). The **equation18** and **19** can be simplified as

$$W_f = \frac{L}{L} Mg\cos(\alpha) - \frac{h_g}{L}\left(F_w + F_g + Mgf_r \frac{r_{dyn}}{h_g}\cos(\alpha) + M\frac{dV}{dt} \right) \quad (20)$$

and

$$W_r = \frac{L}{L} Mg\cos(\alpha) - \frac{h_g}{L}\left(F_w + F_g + Mgf_r \frac{r_{dyn}}{h_g}\cos(\alpha) + M\frac{dV}{dt} \right) \quad (21)$$

Using **equation 5, 17**, **20** and **21** can be rewritten as

$$W_r = \frac{L_a}{L} Mg\cos(\alpha) - \frac{h_g}{L}\left(\left(- \frac{r_{dyn}}{h_g} \right)\right) \quad (22)$$

$$W_r = \frac{L_a}{L} Mg\cos(\alpha) + \frac{h_g}{L}\left(\left(- \frac{r_{dyn}}{h_g} \right)\right) \quad (23)$$

The first term on the right hand side of **equation 22** and **equation 23** is the static load on the front and the rear axles when the vehicle is at rest on level ground. The second term is the dynamic component of the normal load.

The maximum tractive effort (**F_{tmax}**) that the tire-ground contact can support is described by the product of the normal load and the coefficient of road adhesion (**m**). In **Table 3**, the values of coefficient of adhesion are given for different speeds of the vehicle and different road conditions. For the front wheel drive vehicle, **F_{tmax}** is given by

$$F_{t\max} = \mu W_f = \mu \left[\frac{L_b}{L} Mg\cos(\alpha) - \frac{h_g}{L}\left(F_{t\max} - F_r \left(1 - \frac{r_{dyn}}{h_g} \right) \right) \right] \quad (24)$$

$$F_{t\max} = \frac{\mu Mg\cos(\alpha)\left[L_b + f_r \left(h_g - r_{dyn} \right) \right] / L}{1 + \mu h_g / L} \quad (25)$$

For the rear wheel drive vehicle, $\mathbf{F_{tmax}}$ is given by

$$F_{t\max} = \mu W_r = \mu \left[\frac{L_a}{L} Mg\cos(\alpha) + \frac{h_g}{L}\left(F_{t\max} - F_r\left(1 - \frac{r_{dyn}}{h_g}\right)\right)\right] \quad (26)$$

$$F_{t\max} = \frac{\mu Mg\cos(\alpha)\left[L_a - f_r\left(h_g - r_{dyn}\right)\right]/L}{1 - \mu h_g / L} \quad (27)$$

Table 3: Coefficient of road adhesion

Road speed [km/h]	Coefficient of road adhesion for dry roads	Coefficient of road adhesion for wet roads
50	0.85	0.65
90	0.8	0.6
130	0.75	0.55

Adhesion, Dynamic wheel radius and slip

When the tractive effort of a vehicle exceeds the maximum tractive effort limit imposed by the adhesive capability between the tyre and ground, the driven wheels will spin on the ground. The adhesive capability between the tyre and the ground is the main limitation of the vehicle performance especially when the vehicle is driven on wet, icy, snow covered or soft soil roads.

The maximum tractive effort on the driven wheels, transferred from the power plant through the transmission should not exceed the maximum values given by **equation 25** and **equation 27**. Otherwise, the driven wheels will spin on the ground, leading to vehicle instability. The slip between the tyres and the surface can be described as:

$$drive\ slip\ S_T = \frac{\omega_R r_{dyn} - V}{\omega_R r_{dyn}} \qquad (28)$$

where

ω_R = angular speed of the tyre [rad / s]

The dynamic wheel radius (**r_{dyn}**) is calculated from the distance travelled per revolution of the wheel, rolling without slip. The dynamic wheel radius is calculated from a distance travelled at 60km/h. The increasing tyre slip at higher speeds roughly offsets the increase in **r_{dyn}**. The values of **r_{dyn}** for different tyre sizes are given in table 4.

Table 4: Dynamic wheel radius of common tyre sizes

Tyre Size	**Rolling Circumference [m]**	**R_{dyn} [m]**	**Tyre Size**	**Rolling Circumference [m]**	**Rdyn [m]**
Passenger cars			**Passenger cars**		
135 R 13	1.67	0.266	205/65 R15	1.975	0.314
145 R 13	1.725	0.275	195/60 R15	1.875	0.298
155 R 13	1.765	0.281	205/60 R 15	1.91	0.304
145/70 R 13	1.64	0.261	**Light commercial vehicles**		
155/70 R13	1.68	0.267	185 R 14	1.985	0.316
165/70 R 13	1.73	0.275	215 R 14	2.1	0.334
175/70 R 13	1.77	0.282	205 R 14	2.037	0.324

175 R 14	1.935	0.308	195/75 R 16	2.152	0.343
185 R 14	1.985	0.316	205/75 R 16	2.2	0.35
195/70 R 14	1.94	0.309	**Trucks and buses**		
185/65 R 14	1.82	0.29	12 R 22.5	3.302	0.526
185/60 R 14	1.765	0.281	315/80 R 22.5	3.295	0.524
195/60 R	1.8	0.286	295/80 R	3.215	0.512

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14			22.5		
195/70 R 15	2	0.318	215/75 R 17.5	2.376	0.378
185/65 R15	1.895	0.302	275/70 R 22.5	2.95	0.47
195/65 R15	1.935	0.308	305/70 R 19.5	2.805	0.446

References:

[1] M. Ehsani, *Modern Electric, Hybrid Electric and Fuel Cell Vehicles: Fundamentals, Theory and Design*, CRC Press, 2005

Suggested Reading:

[1] I. Husain, *Electric and Hybrid Electric Vehicles*, CRC Press, 2003

[2] C. C. Chan and K. T. Chau, *Modern Electric Vehicle Technology*, Oxford Science Publication, 2001

[3] G. Lechner and H. Naunheimer, *Automotive Transmissions: Fundamentals, Selection, Design and Application*, Springer, 1999

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Lecture 4: Vehicle Power Plant and Transmission Characteristics

Vehicle Power Plant and Transmission Characteristics

Introduction

The topics covered in this chapter are as follows:

- The drive train configuration
- Various types of vehicle power plants
- The need of gearbox in a vehicle
- The mathematical model of vehicle performance

Drive train Configuration

An automotive drive train is shown in **Figure 1**. It consists of:

- a power plant
- a clutch in a manual transmission or a torque converter in automatic transmission
- a gear box
- final drive
- differential shaft
- driven wheels

The torque and rotating speed from the output shaft of the power plant are transmitted to the driven wheels through the clutch or torque converter, gearbox, final drive, differential and drive shaft.

The clutch is used in manual transmission to couple or decouple the gearbox to the power plant. The torque converter in an automatic transmission is hydrodynamic device, functioning as the clutch in manual transmission with a continuously variable gear ratio.

The gearbox supplies a few gear ratios from its input shaft to its output shaft for the power plant torque-speed profile to match the requirements of the load. The final drive is usually a pair of gears that supply a further speed reduction and distribute the torque to each wheel through the differential.

Clutch

Driven wheel

1 3 5

Vehicle

Differential

Drive Shaft

Powe r Plant

2 4 R

Driven wheel

Transmission

Figure 1: An automobile power train

Vehicle power plant

There are two limiting factors to the maximum tractive effort of the vehicle:

- Maximum tractive effort that the tire-ground contact can support
- Tractive effort that the maximum torque of the power plant can produce with the given driveline gear ratios.

The smaller of these factors will determine the performance potential of the vehicle. Usually it is the second factor that limits the vehicles performance.

The classification of various types of power plants used in a vehicle is shown in **Figure 2**.

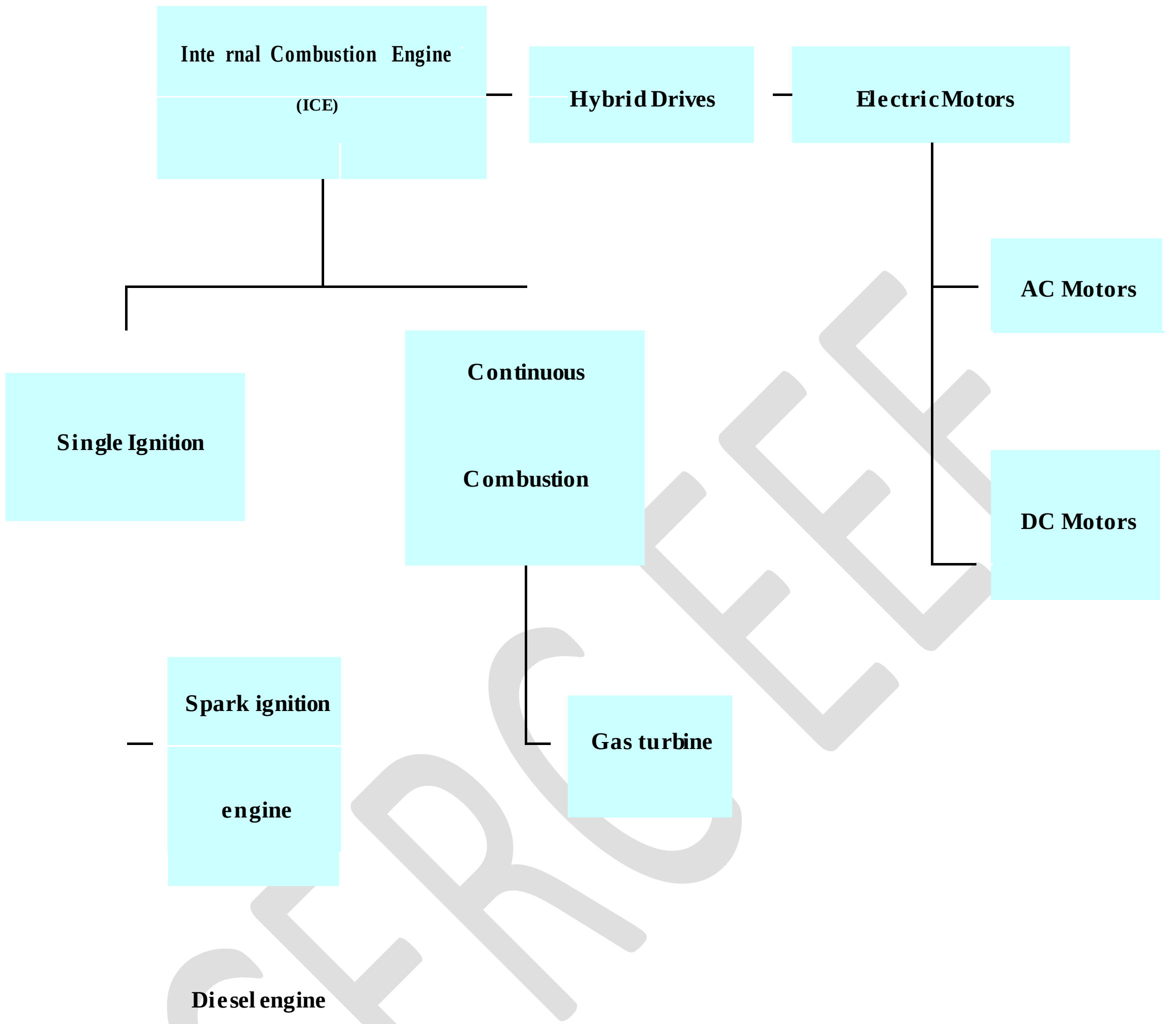

Figure 2: Classification of vehicle power plat

In selecting a suitable power plant, the following factors are considered:

- ***Operating performance***
- ***Economy***
- ***Environment friendliness***

For vehicular applications, the ideal performance characteristic of a power plant is constant power output over the full speed range. Consequently, the torque varies hyperbolically with respect to speed as shown in **Figure 3**. This ideal performance characteristic of the power plant will ensure that the maximum power is available at any vehicle speed, thus resulting in optimal vehicle performance. In practice however, the torque is constrained to be constant a low speeds. This is done so as not to be over the maxima limited by the adhesion between the tyre-ground contact areas. The **internal combustion** (**IC**) engines are the most commonly used power plants for the land vehicles. In hybrid and electric vehicle technology, the **electric motor** is used.

Torque

Powe r

Speed

Figure 3: Ideal performance characteristics for a vehicle power plant

Internal combustion engine

The internal combustion engines used in the vehicles are based on two principles:

- **spark ignition** (**petrol engines**) principle
- **Diesel** principle.

The key features of the ICs based spark ignition principle are:

- high power/weight ratio
- good performance
- low combustion noise.

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The disadvantages of are the ICs based spark ignition principle are:

- quality of fuel required
- higher fuel consumption.

The advantages of the diesel engines are:

- low fuel consumption
- low maintenance requirement due to absence of ignition system
- low fuel quality required

The disadvantages of the diesel engine are

- high level of particulate emission

P_{max}

- greater weight and higher price
- higher levels of noise

The two typical characteristic curves used to describe the engine characteristic are:

- torque vs. engine speed curve at full load (100% acceleration pedal position)
- power vs. engine speed curve at full load (100% acceleration pedal position)

These two characteristic curves are shown in **Figure 4**. In **Figure 4** the following nomenclature is used:

$= P_n$ = Maximum engine power = Nominal power

$P(T_{max})$ = Engine power at maximum torque

T_{max} = Maximum engine torque

$T(P_{max}) = T_n$ = Engine at maximum power = Nominal Torque

$n\,(P_{max}) = n_n$ = Engine speed at maximum power = Nominal
speed $n\,(T_{max})$ = Engine speed at maximum torque

Various indices are used to facilitate comparison between different types of engine. The two most important indices are:

- **torque increase (torque elasticity)** defined as

$$\tau = \frac{T_{max}}{T_n} \qquad (1)$$

where

T_{max} = maximum engine torque

T_n = engine torque at maximum power, also known as nominal torque

.

- **engine speed ratio** defined as

$$\nu = \frac{n_n}{n(T_{\max})} \qquad (2)$$

where

n_n = *engine speed at maximum power, also known as nominal speed*
$n(T_{\max})$ = *engine speed at maximum torque*

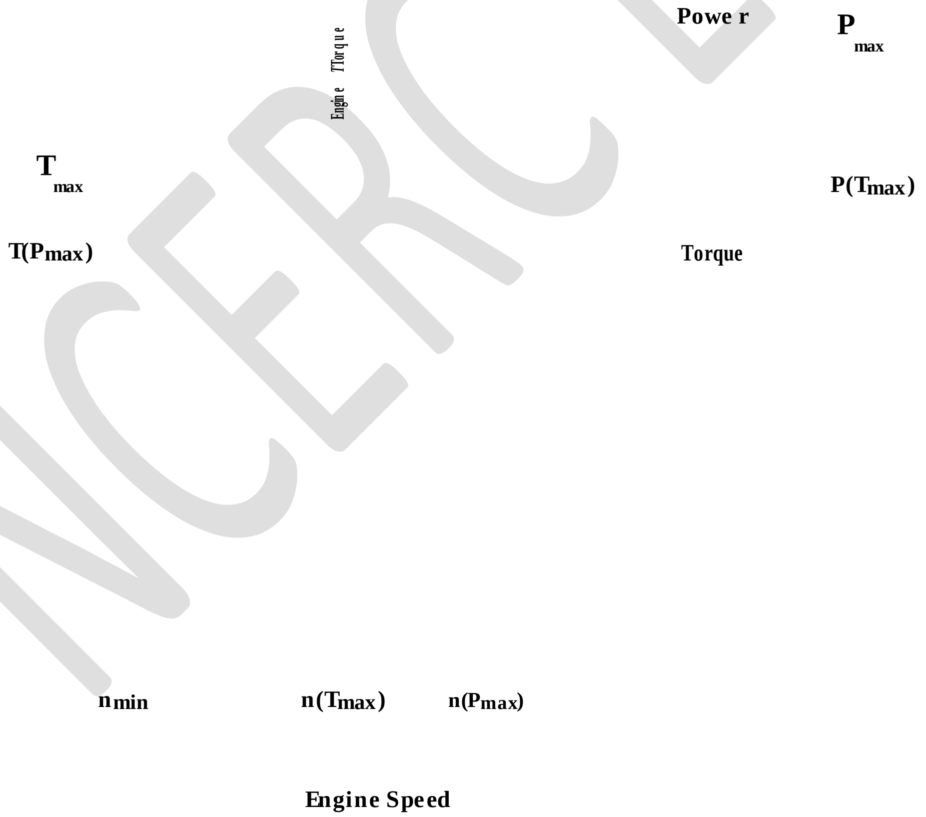

Figure 4: Characteristic curves of an internal combustion engine

The higher value of the product **τν** better engine power at low and medium engine speeds.

This in turn means less frequent gear changing.

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Electric Motor

The electric motors have are ideal for vehicle application because of the torque speed characteristics of the motors (**Figure 5**). Electric motors are capable of delivering a high starting torque. It is very important to select proper type of motor with a suitable rating. For example, it is not accurate to simply refer to a 10 h.p. motor or a 15 h.p. motor, because horsepower varies with volts and amps, and peak horsepower is much higher than the continuous rating.

Power

Torque

Speed

Figure 5: Torque vs. speed and power vs. speed characteristics of electric motor

It is also confusing to compare electric motors to IC engines, since electric motors are designed for a continuous rating under load and IC engines are rated at their peak horsepower under loaded condition. The commonly used motors in EVs are:

- AC motors
- Permanent magnet (PM) motors
- Series wound DC motors
- Shunt wound DC motors

The DC series motors were used in a number of prototype Electric Vehicle (EVs) and prior to that mainly due to the ease of control. However, the size and maintenance requirements of DC motors are making their use obsolete. The recent EVs and Hybrid Electric Vehicles (HEVs) use AC, PM and Switched Reluctance motors. A classification of motors used in EVs is shown in **Figure 6**.

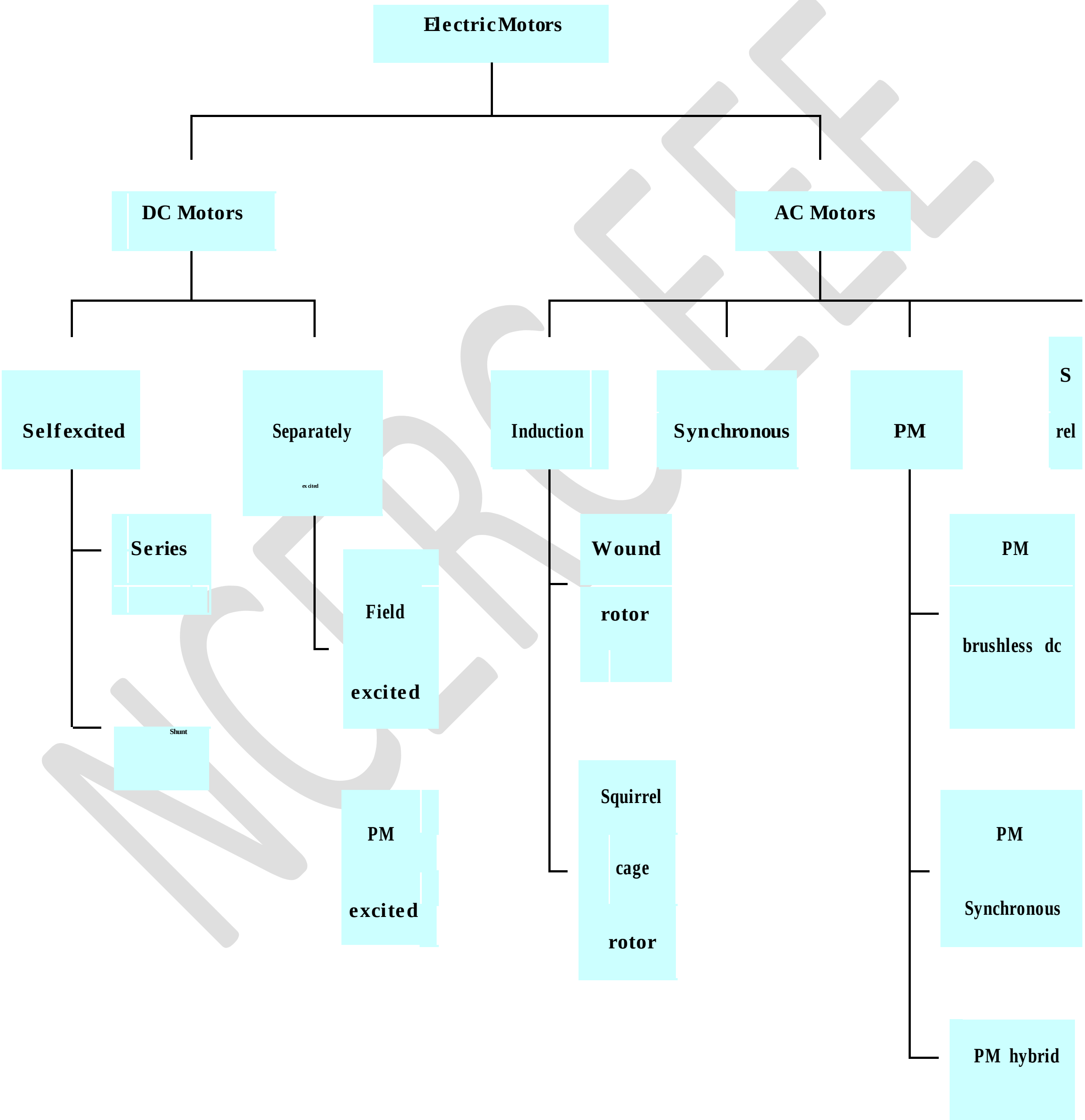

Figure 6: Classification of electric motors used in EVs

The AC Induction Motor (IM) technology is very mature and significant research and development activities have taken place in the area of induction motor drives. The control of

IM is more complex than DC motors, but the availability of fast digital processors, computational complexity can easily be managed. The competitor to the induction motor is the permanent magnet (PM) motor. The permanent magnet motors have magnets on the rotor, while the stator construction is same as that of induction motor. The PM motors can be surface mounted type or the magnets can be inset within the rotor. The PM motors can also be classified as sinusoidal type or trapezoidal type depending on the flux density distribution in the air gap. Permanent magnet motors with sinusoidal air gap flux distribution are called Permanent Magnet synchronous Motors (PMSM) and the with trapezoidal air gap flux distribution are called Brushless DC (BLDC) motors.

The need for gearbox

Internal combustion engines today drive most of the automobiles. These internal combustion engines work either on the principle of spark ignition or diesel principle. In addition to the many advantages of the internal combustion engine, such as high power to weight ratio and relatively compact energy storage, it has two fundamental disadvantages:

i. ***Unlike the electric motors, the internal combustion engine cannot produce torque at zero speed.***

ii. ***The internal combustion engine produces maximum power at a certain engine speed***.

iii. The efficiency of the engine, i.e. its fuel consumption, is very much dependent on the operating point in the engine's performance map.

With a maximum available engine power P_{max} and a road speed of v, the ***ideal traction hyperbola*** F_{ideal} and the ***effective traction hyperbola*** F_{effec} can be calculated as follows:

$$F_{ideal} = \frac{P_{max}}{v}$$

$$F_{ideal} = \frac{P_{max}}{v}\eta_{tot}$$

where

η_{tot} = efficiency of the drivetrain

(1)

Hence, if the full load engine power P_{max} were available over the whole speed range, the traction hyperbolas shown in **Figure 7** would result. However, the P_{max} is not available for the entire speed range. The actual traction profile of the ICE (F_{engine}) is shown in **Figure 7**. From **Figure 7** it is evident that the entire shaded area cannot be used.

Without drive train efficiency:Ideal

Adhesion Limit

traction hyperbola F_{ideal}

With drive train efficiency:Ideal

traction hyperbola F_{effec}

Internal combustion engine traction available F_{engine}

Speed

Figure 7: Traction force vs. speed map of an internal combustion engine without gearbox

In order to utilize the shaded area, shown in **Figure 7**, additional output converter is required. The output converter must convert the characteristics of the combustion engine in such a way that it approximates as closely as possible to the ideal ***traction hyperbola*** (**Figure 8)**.

Adhesion Limit

Traction force available

in second gear

Traction required for 0% gradient

1st

gear

2nd

gear

3rd

gear

4th

gear

Speed

Figure 8: Traction force vs. speed map of an internal combustion engine with gearbox

The proportion of the shaded area, i.e. the proportion of impossible driving states, is significantly smaller when an output converter is used. Thus, the power potential of the engine is better utilized. The Figure 8 shows how increasing the number of gears gives a better approximation of the *effective traction hyperbola*.

Drive train tractive effort and vehicle speed

After having dealt with the configuration of the drivetrain, this section deals with the ***tractive effort***. The torque transmitted from the power plant to the driven wheels (**T_W**) is given by:

$$T_w = i_g \, i_o \eta_t \, T_p$$

where

i_g = *gear ratio of the transmission* (1)

i_o = *gear ratio of the fianl drive*

η_t = *efficiency of the driveline from the power plant to the driven wheels*

T_p = *torque output from the power plant* $[Nm]$

The tractive effort on the driven wheels (**Figure 9**) is expressed as

$$F_t = \frac{T_w}{r_{dyn}}$$

where (2)

r_{dyn} = *dynamic radius of the tyre* $[m]$

Substituting value of $\mathbf{T_W}$ from **equation 1** into **equation 2** gives

$$F_t = \frac{T_p\, i_g\, i_0 \eta_t}{r_{dyn}} \qquad (3)$$

The total mechanical efficiency of the transmission between the engine output shaft and driven wheels is the product of the efficiencies of all the components of the drive train. The rotating speed of the driven wheel is given by

$$N_w = \frac{N_p}{i_g\, i_0} \; [rpm] \qquad (4)$$

where

N_p = *rottational speed of the transmission* [*rpm*]

The rotatational speed $\mathbf{N_p}$ of the transmission is equal to the engine speed in the vehicle with a manual transmission and the turbine speed of a torque converter in the vehicle with an automatic transmission. The ***translation speed of the wheel*** (***vehicle speed***) is expressed as

$$V = \frac{\pi N_w\, r_{dyn}}{30} \; [m/s] \qquad (5)$$

By substituting the value of $\mathbf{N_W}$ from **equation 4** into **equation 5**, the ***vehicle speed*** can be expressed as

$$V = \frac{\pi N_p\, r_{dyn}}{30 i_g\, i_o} \; [m/s] \qquad (6)$$

Vehicle performance

The performance of a vehicle is determined by the following factors:

- maximum cruising speed
- gradeability
- acceleration

.

Maximum Cruising Speed

The maximum speed of a vehicle is defined as the constant cruising speed that the vehicle can achieve with full power plant load on a flat road. The maximum speed of a vehicle is determined by the equilibrium between the tractive effort of the vehicle and the resistance and maximum speed of the power plant and gear ratios of the transmission. This equilibrium is:

$$\frac{T_p i_g i_0 \eta_t}{r_{dyn}} = Mgf_r \cos(\alpha) + \frac{1}{2}\rho_a C_D A_f V^2 \tag{30}$$

where

i_g = *gear ratio of the transmission*

i_0 = *gear ratio of the final drive*

η_t = *efficiency of the driveline from the power plant to the driven wheels*

T_p = *torque output of the power plant* [*Nm*]

equation 30 shows that the vehicle reaches its maximum speed when the tractive effort, represented by the left hand side term, equals the resistance, represented by the right hand side. The intersection of the tractive effort curve and the resistance curve is the maximum speed of the vehicle, **Figure 9**.

0

120

100

80

60

40

20

rt

$F_r + F_w + F_g$

Resistance on grade

Tractive effo

0 50 100 150 200 250

Speed [km/h]

Figure 9: Tractive effort of an electric motor powered vehicle with a single speed transmission and its resistance

For some vehicles, no intersection exists between the tractive effort curve and the resistance curve, because of a large power plant. In such a case the maximum speed of the vehicle is determined by the maximum speed of the power plant. This maximum speed is given by

$$V=\frac{\pi n_{p\max} r_{dyn\max}}{30 i_0 i_{g\min}} \tag{31}$$

where

$i_{g\min}$ = min *imum gear ratio of the transmission*

i_0 = *gear ratio of the final drive*

$n_{p\max}$ = max *imum speed of the power plant* (*motor or engine*)[*rpm*]

T_p = *torque output of the power plant* [*Nm*]

r_{dyn} = *dynamic radius of the tyre* [*m*]

Gradeability

Gradeability is defined as the grade angle that the vehicle can negotiate at a certain constant speed. For heavy commercial vehicles the gradeability is usually defined as the maximum grade angle that the vehicle can overcome in the whole speed range.

When the vehicle is driving on a road with relatively small grade and constant speed, the tractive effort and resistance equilibrium can be expressed as

$$\frac{T_p i_0 i_g \eta_t}{r_{dyn}} = Mgf_r + \frac{1}{2}\rho_a C_D A_f V^2 + Mgi \tag{32}$$

Hence,

$$i = \frac{T_p i_0 i_g \eta_t / r_{dyn} - Mgf_r - 1/2 \rho_a C_D A_f V^2}{Mg} = d - f_r \tag{33}$$

where

$$T i i \eta / r \quad -1/2\rho C_D A \; V^2 \tag{34}$$

$$d = \frac{\quad p\,0\,g \quad t \quad dyn \quad \quad f}{Mg}$$

The factor **d** is called the performance factor. When the vehicle drives on a road with a large grade, the gradeability of the vehicle can be calculated as

[redacted]

$$\sin(\alpha) = \frac{d - f_r^2 \quad 1 - d^2 + f_r^2}{1 + f_r^2} \qquad (35)$$

Acceleration Performance

The acceleration of a vehicle is defined by its acceleration time and distance covered from zero speed to a certain high speed on a level ground. The acceleration of the vehicle can be expressed as

$$a = \frac{dV}{dt} = \frac{F_t - F_f - F_w}{M\delta} = \frac{T_p i_0 i_g \eta_t / r_{dyn} - Mgf_r - 1/2\rho_a C_D A_f V^2}{M\delta} = \frac{g}{\delta}(d - f_r) \qquad (36)$$

where δ is the rotational inertia factor taking into account the equivalent mass increase due to the angular moments of the rotating components. This mass factor can be written as

$$\delta = 1 + \frac{I_w}{Mr_{dyn}^2} + \frac{i_0^2 i_g^2 I_p}{Mr_{dyn}^2} \qquad (37)$$

I_w = *total angular inertial moment of the wheels*

I_p = *total angular inertial moment of the rotating components associated with the power plant*

To determine the value of δ, it is necessary to determine the values of the mass moments of inertia of all the rotating parts. In case the mass moments of inertia are not available then, the rotational factor (δ) can be approximated as:

$$\delta = 1 + \delta_1 + \delta_2 i_g^2 i_0^2 \qquad (38)$$

$$\delta_1 \approx 0.04$$

$$\delta_2 \approx 0.0025$$

The acceleration rate along with vehicle speed for a petrol engine powered vehicle with a four gear transmission and an electric motor powered vehicle with a single gear transmission are shown in **Figure 10** and **Figure 11** respectively.

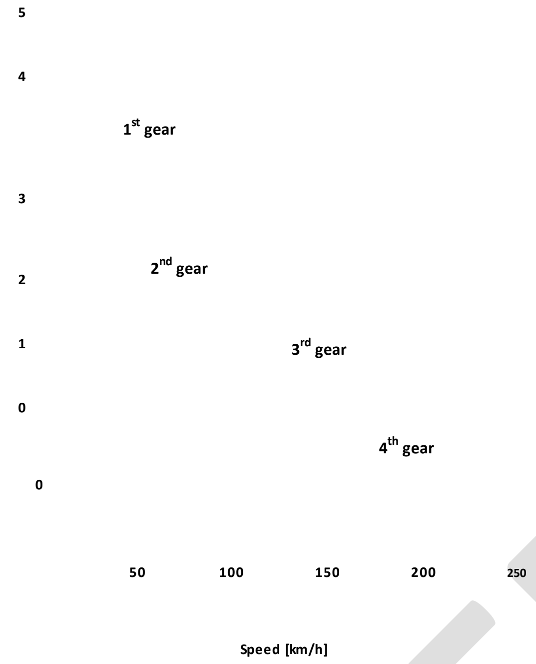

Figure 10: Acceleration rate of a petrol engine powered vehicle with four gears

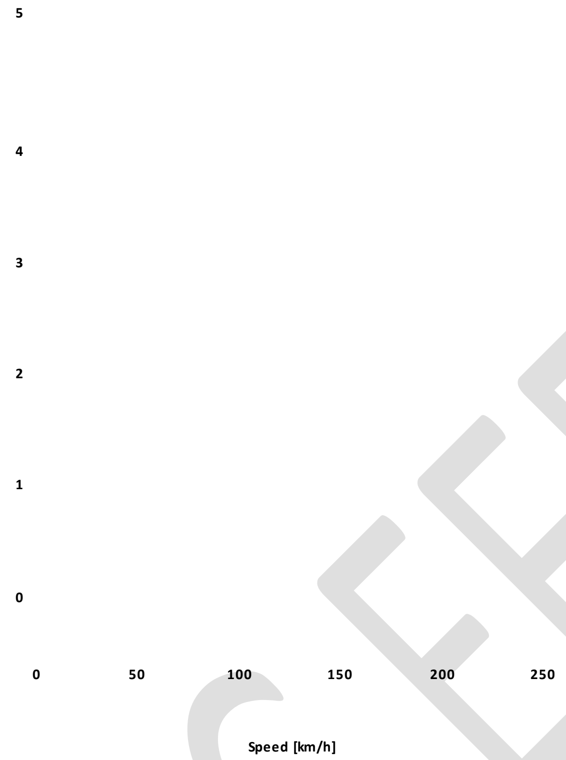

Figure 11: Acceleration rate of an electric machine powered vehicle with a single gear

From **equation 36**, the acceleration time $\mathbf{t_a}$ and distance $\mathbf{S_a}$ from a lower speed $\mathbf{V_1}$ to a higher speed $\mathbf{V_2}$ can be expressed as

$$t_a = \int_{V_1}^{V_2} \frac{M\delta}{T_p i_g i_0 \eta_t / r_{dyn} - Mgf_r - 1/2\rho_a C_D A_f V^2} dV \tag{39}$$

and

$$S_a = \int_{V_1}^{V_2} \frac{M\delta V}{T_p i_g i_0 \eta_t / r_{dyn} - Mgf_r - 1/2\rho_a C_D A_f V^2} dV \tag{40}$$

The power plant torque $\mathbf{T_p}$ in **equation 39** and **equation 40** is a function of speed of the power plant. The speed of the power plant is in turn a function of the vehicle speed and gear ratio of the transmission. Hence, analytical solution of **equation 39** and **equation 40** is not possible. Numerical methods are usually used to solve these equations.

Suggested Reading

[1] I. Husain, *Electric and Hybrid Electric Vehicles*, CRC Press, 2003

[2] G. Lechner and H. Naunheimer, *Automotive Transmissions: Fundamentals, Selection, Design and*

Application, Springer, 1999

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Module 3: Architecture of Hybrid and Electric Vehicles

Lecture 5: Basic Architecture of Hybrid Drive Trains and Analysis of Series Drive Train

Basic Architecture of Hybrid Drive Trains and Analysis of Series Drive Train

Introduction

The topics covered in this chapter are as follows:

λ. Hybrid Electric Vehicles (HEV)

μ. Energy use in conventional vehicles

ν. Energy saving potential of hybrid drive trains

ο. Various HEV configurations and their operation modes

The Hybrid Electric Vehicle (HEV)

What exactly is an HEV? The definition available is so general that it anticipates future technologies of energy sources. The term ***hybrid vehicle*** refers to a vehicle with at least two sources of power. A ***hybrid-electric vehicle*** indicates that ***one source of power*** is provided by an ***electric motor***. The ***other source of motive power*** can come from a number of different technologies, but is typically provided by an ***internal combustion engine*** designed to run on either gasoline or diesel fuel. As proposed by Technical Committee (Electric Road Vehicles) of the International Electrotechnical Commission, ***an HEV is a vehicle in which propulsion energy is available from two or more types of energy sources and at least one of them can deliver electrical energy***. Based on this general definition, there are many types of HEVs, such as:

- the gasoline ICE and battery
- diesel ICE and battery
- battery and FC
- battery and capacitor
- battery and flywheel
- battery and battery hybrids.

Most commonly, the propulsion force in HEV is provided by a combination of electric motor and an ICE. The electric motor is used to improve the energy efficiency (improves fuel consumption) and vehicular emissions while the ICE provides extended range capability.

Energy Use in Conventional Vehicles

In order to understand how a HEV may save energy, it is necessary first to examine how conventional vehicles use energy. The breakdown of energy use in a vehicle is as follows:

δ In order to maintain movement, vehicles must produce power at the wheels to overcome:

- aerodynamic drag (air friction on the body surfaces of the vehicle, coupled with pressure forces caused by the air flow)
- rolling resistance (the resistive forces between tires and the road surface)
- resistive gravity forces associated with climbing a grade

N Further, to accelerate, the vehicle must its inertia. Most of the energy expended in acceleration is then lost as heat in the brakes when the vehicle is brought to a stop.

O The vehicle must provide power for accessories such as heating fan, lights, power steering, and air conditioning.

Π Finally, a vehicle will need to be capable of delivering power for acceleration with very little delay when the driver depresses the accelerator, which may necessitate keeping the power source in a standby (energy-using) mode.

A conventional engine-driven vehicle uses its engine to translate fuel energy into shaft power, directing most of this power through the drivetrain to turn the wheels. Much of the heat generated by combustion cannot be used for work and is wasted, both because heat engines have theoretical efficiency limit. Moreover, it is impossible to reach the theoretical efficiency limit because:

- α some heat is lost through cylinder walls before it can do work
- α some fuel is burned at less than the highest possible pressure
- α fuel is also burned while the engine is experiencing negative load (during braking) or when the vehicle is coasting or at a stop, with the engine idling.

Although part of engine losses would occur under any circumstances, part occurs because in conventional drivetrains, engines are sized to provide very high levels of peak power for the acceleration capability expected by consumers, about 10 times the power required to cruise at 100Km/h. However, the engines are operated at most times at a small fraction of peak power and at these operating points they are quite inefficient.

Having such a large engine also increases the amount of fuel needed to keep the engine operating when the vehicle is stopped or during braking or coasting, and increases losses due to the added weight of the engine, which increases rolling resistance and inertial losses. Even gradeability requirements (example: 55 mph up a 6.5% grade) require only about 60 or 70% of the power needed to accelerate from 0 to 100Km/h in under 12 seconds.

The **Figure 1** shows the translation of fuel energy into work at the wheels for a typical midsize vehicle in urban and highway driving. From **Figure 1** it can be observed that:

Λ. At best, only 20% of the fuel energy reaches the wheels and is available to overcome the tractive forces, and this is on the highway when idling losses are at a minimum, braking loss is infrequent, and shifting is far less frequent.

M. Braking and idling losses are extremely high in urban driving and even higher in more congested driving, e.g., within urban cores during rush hour. Braking loss represents 46% of all tractive losses in urban driving. Idling losses represent about one sixth of the fuel energy on this cycle.

N. Losses to aerodynamic drag, a fifth or less of tractive losses in urban driving, are more than half of the tractive losses during highway driving.

Figure 1:Translation of fuel energy into work in a vehicle

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Energy Savings Potential of Hybrid Drivetrains

In terms of overall energy efficiency, the conceptual advantages of a hybrid over a conventional vehicle are:

ω) **Regenerative braking.** A hybrid can capture some of the energy normally lost as heat to the mechanical brakes by using its electric drive motor(s) in generator mode to brake the vehicle

ξ) **More efficient operation of the ICE, including reduction of idle.** A hybrid can avoid some of the energy losses associated with engine operation at speed and load combinations where the engine is inefficient by using the energy storage device to either absorb part of the ICE's output or augment it or even substitute for it. This allows the ICE to operate only at speeds and loads where it is most efficient. When an HEV is stopped, rather than running the engine at idle, where it is extremely inefficient, the control system may either shut off the engine, with the storage device providing auxiliary power (for heating or cooling the vehicle interior, powering headlights, etc.), or run the engine at a higher-than-idle (more efficient) power setting and use the excess power (over auxiliary loads) to recharge the storage device. When the vehicle control system can shut the engine off at idle, the drivetrain can be designed so that the drive motor also serves as the starter motor, allowing extremely rapid restart due to the motor's high starting torque.

ψ) **Smaller ICE:** Since the storage device can take up a part of the load, the

HEV's ICE can be down sized. The ICE may be sized for the continuous load and not for the very high short term acceleration load. This enables the ICE to operate at a higher fraction of its rated power, generally at higher fuel

efficiency, during most of the driving.

There are counterbalancing factors reducing hybrids' energy advantage, including:

ω) **Potential for higher weight.** Although the fuel-driven energy source on a hybrid generally will be of lower power and weight than the engine in a conventional vehicle of similar performance, total hybrid weight is likely to be higher than the conventional vehicle it replaces because of the added weight of the storage device, electric motor(s), and other components. This depends, of course, on the storage mechanism chosen, the vehicle performance requirements, and so forth.

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6. **Electrical losses.** Although individual electric drivetrain components tend to be quite efficient for one-way energy flows, in many hybrid configurations, electricity flows back and forth through components in a way that leads to cascading losses. Further, some of the components may be forced to operate under conditions where they have reduced efficiency. For example, like ICEs, most electric motors have lower efficiency at the low-speed, low-load conditions often encountered in city driving. Without careful component selection and a control strategy that minimizes electric losses, much of the theoretical efficiency advantage often associated with an electric drivetrain can be lost.

HEV Configurations

In **Figure 2** the generic concept of a hybrid drivetrain and possible energy flow route is shown. The various possible ways of combining the power flow to meet the driving requirements are:

A powertrain 1 alone delivers power

B powertrain 2 alone delivers power

C both powertrain 1 and 2 deliver power to load at the same time

D powertrain 2 obtains power from load (regenerative braking)

E powertrain 2 obtains power from powertrain 1

F powertrain 2 obtains power from powertrain 1 and load at the same time

G powertrain 1 delivers power simultaneously to load and to powertrain 2

H powertrain 1 delivers power to powertrain 2 and powertrain 2 delivers power ton load

I powertrain 1 delivers power to load and load delivers power to powertrain 2.

Figure 2:Generic Hybrid Drivetrain [1]

The load power of a vehicle varies randomly in actual operation due to frequent acceleration, deceleration and climbing up and down the grades. The power requirement for a typical driving scenario is shown in **Figure 3**. The load power can be decomposed into two parts:

a steady power, i.e. the power with a constant value

b dynamic power, i.e. the power whose average value is zero

Power [kW] Power [kW] Power [kW]

Dynamic Power

Average power

0 = 0 + 0

Time [s] Time [s] Time [s]

Figure 3: Load power decomposition [1]

In HEV one powertrain favours steady state operation, such as an ICE or fuel cell. The other powertrain in the HEV is used to supply the dynamic power. The total energy output from the dynamic powertrain will be zero in the whole driving cycle. Generally, electric motors are used to meet the dynamic power demand. This hybrid drivetrain concept can be implemented by different configurations as follows:

- Ω Series configuration
- Ξ Parallel configuration
- Ψ Series-parallel configuration
- Z Complex configuration

In **Figure 4** the functional block diagrams of the various HEV configurations is shown. From **Figure 4** it can be observed that the key feature of:

- N series hybrid is to couple the ICE with the generator to produce electricity for pure electric propulsion.
- O parallel hybrid is to couple both the ICE and electric motor with the transmission via the same drive shaft to propel the vehicle

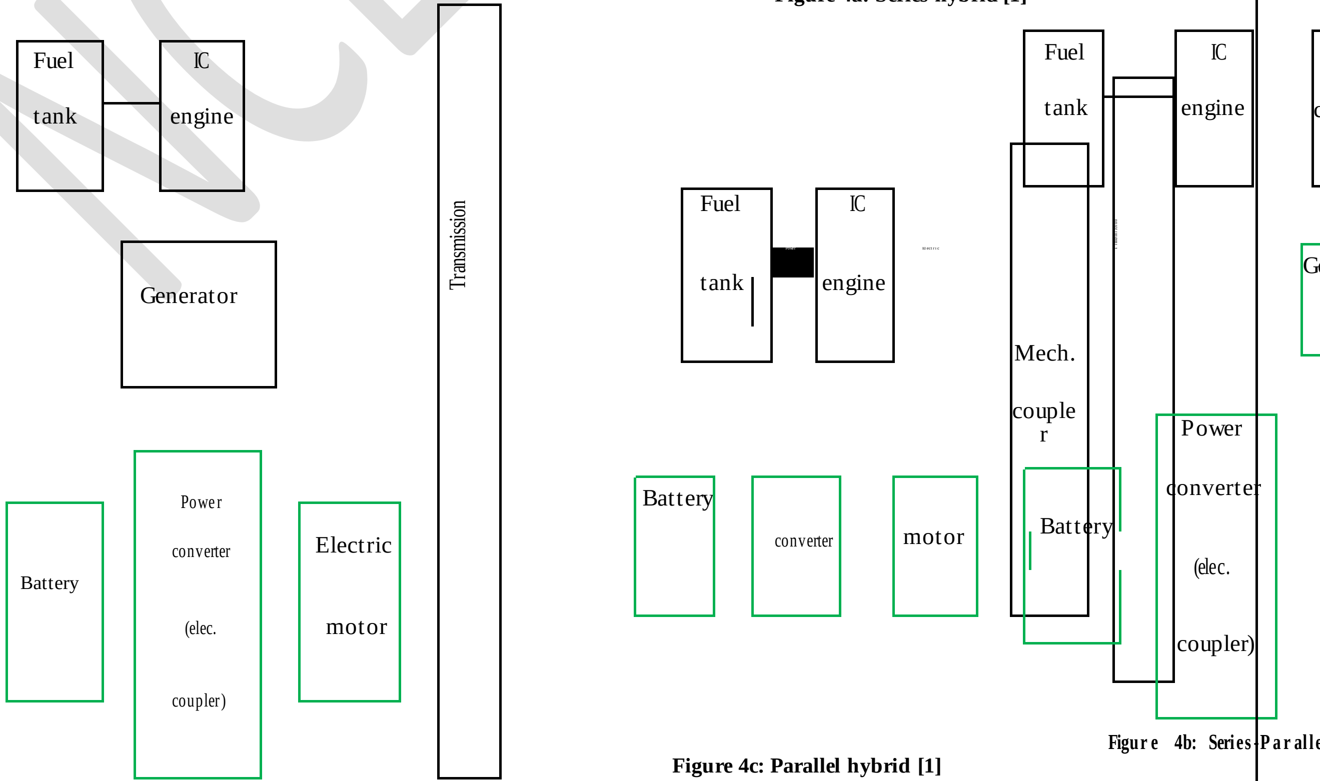

Figure 4a: Series hybrid [1]

Figure 4b: Series-Paralle

Figure 4c: Parallel hybrid [1]

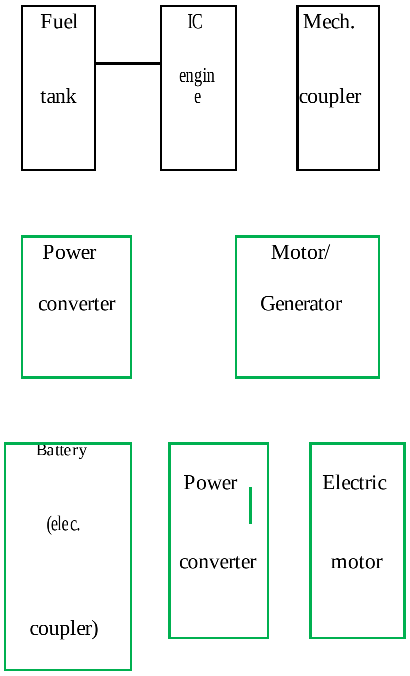

Figure 4d: Complex hybrid [1]

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Series Hybrid System:

In case of series hybrid system (**Figure 4a**) the mechanical output is first converted into electricity using a generator. The converted electricity either charges the battery or can bypass the battery to propel the wheels via the motor and mechanical transmission. Conceptually, it is an ICE assisted Electric Vehicle (EV). The advantages of series hybrid drivetrains are:

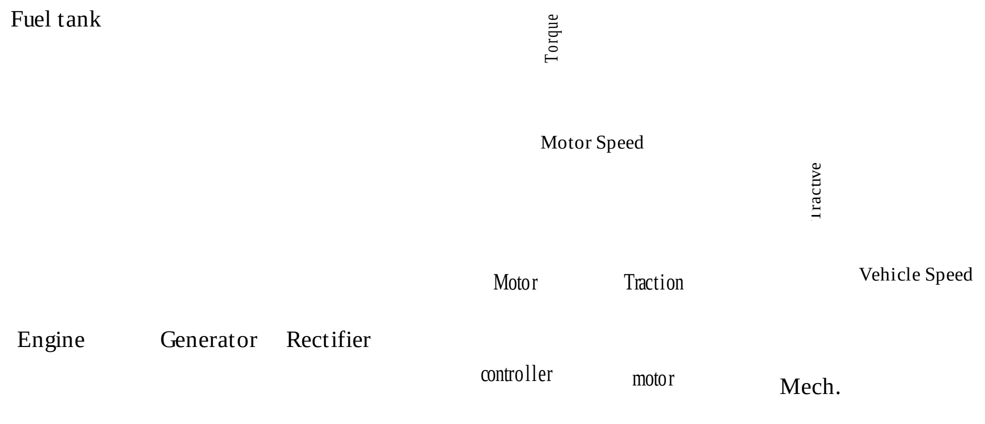
- mechanical decoupling between the ICE and driven wheels allows the IC engine operating at its very narrow optimal region as shown in **Figure 5**.
- nearly ideal torque-speed characteristics of electric motor make multigear transmission unnecessary.

However, a series hybrid drivetrain has the following disadvantages:

= the energy is converted twice (mechanical to electrical and then to mechanical) and this reduces the overall efficiency.

= Two electric machines are needed and a big traction motor is required because it is the only torque source of the driven wheels.

The series hybrid drivetrain is used in heavy commercial vehicles, military vehicles and buses. The reason is that large vehicles have enough space for the bulky engine/generator system.



Trans.

DC

Engine

Operating

Engine Speed

region

DC

Battery

pack

Electrical

coupler

Traction

Battery

Battery charge

charger

Figure 5: Detailed Configuration of Series Hybrid Vehicle [1]

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Parallel Hybrid System:

The parallel HEV (**Figure 4b**) allows both ICE and electric motor (EM) to deliver power to drive the wheels. Since both the ICE and EM are coupled to the drive shaft of the wheels via two clutches, the propulsion power may be supplied by ICE alone, by EM only or by both ICE and EM. The EM can be used as a generator to charge the battery by regenerative braking or absorbing power from the ICE when its output is greater than that required to drive the wheels. The advantages of the parallel hybrid drivetrain are:

= both engine and electric motor directly supply torques to the driven wheels and no energy form conversion occurs, hence energy loss is less

= compactness due to no need of the generator and smaller traction motor.

[2] mechanical coupling between the engines and the driven wheels, thus the engine operating points cannot be fixed in a narrow speed region.

[3] The mechanical configuration and the control strategy are complex compared to series hybrid drivetrain.

Due to its compact characteristics, small vehicles use parallel configuration. Most passenger cars employ this configuration.

Series-Parallel System

In the series-parallel hybrid (**Figure 4c**), the configuration incorporates the features of both the series and parallel HEVs. However, this configuration needs an additional electric machine and a planetary gear unit making the control complex.

Complex Hybrid System

The complex hybrid system (**Figure 4d**) involves a complex configuration which cannot be classified into the above three kinds. The complex hybrid is similar to the series-parallel hybrid since the generator and electric motor is both electric machines. However, the key difference is due to the bi-directional power flow of the electric motor in complex hybrid and the unidirectional power flow of the generator in the series-parallel hybrid. The major disadvantage of complex hybrid is higher complexity.

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References:

[4] M. Ehsani, *Modern Electric, Hybrid Electric and Fuel Cell Vehicles: Fundamentals, Theory and Design*, CRC Press, 2005

Suggested Reading:

- I. Husain, *Electric and Hybrid Electric Vehicles*, CRC Press, 2003

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Lecture 6: Power Flow in HEVs

Power Flow in HEVs

Introduction

The following topics are covered in this lecture

- Power Flow Control
- Power Flow Control in Series Hybrid
- Power Flow Control in Parallel Hybrid
- Power Flow Control in Series-Parallel Hybrid

Power Flow Control

Due to the variations in HEV configurations, different power control strategies are necessary to regulate the power flow to or from different components. All the control strategies aim satisfy the following goals:

- maximum fuel efficiency
- minimum emissions
- minimum system costs
- good driving performance

The design of power control strategies for HEVs involves different considerations such as:

- ***Optimal ICE operating point:*** The optimal operating point on the torque-speed plane of the ICE can be based on maximization of fuel economy, the minimization of emissions or a compromise between fuel economy and emissions.

- ***Optimal ICE operating line:*** In case the ICE needs to deliver different power demands, the corresponding optimal operating points constitute an optimal operating line.

- ***Safe battery voltage:*** The battery voltage may be significantly altered during discharging, generator charging or regenerative charging. This battery voltage should not exceed the maximum voltage limit nor should it fall below the minimum voltage limit.

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Power Flow Control in Series Hybrid

In the series hybrid system there are four operating modes based on the power flow:

- **Mode 1:** During startup (**Figure 1a**), normal driving or acceleration of the series HEV, both the ICE and battery deliver electric energy to the power converter which then drives the electric motor and hence the wheels via transmission.

- **Mode 2:** At light load (**Figure 1b**), the ICE output is greater than that required to drive the wheels. Hence, a fraction of the generated electrical energy is used to charge the battery. The charging of the batter takes place till the battery capacity reaches a proper level.

- **Mode 3:** During braking or deceleration (**Figure 1c**), the electric motor acts as a generator, which converts the kinetic energy of the wheels into electricity and this, is used to charge the battery.

- **Mode 4:** The battery can also be charged by the ICE via the generator even when the vehicle comes to a complete stop (**Figure 1d**).

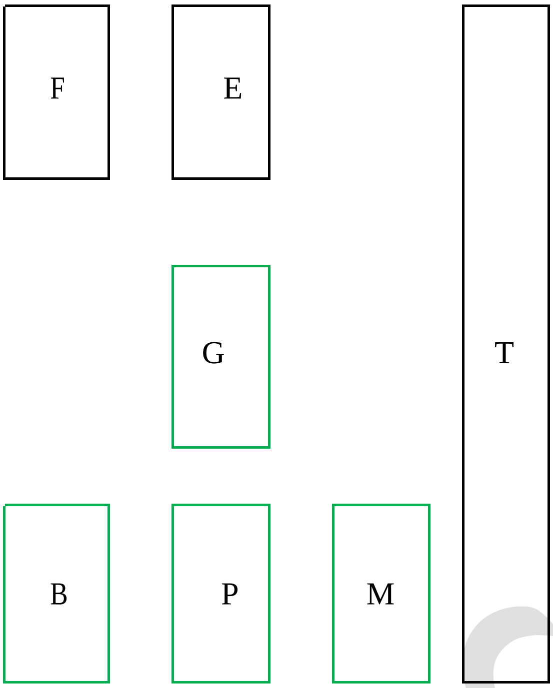


Figure 1a: Mode 1, normal driving or acceleration

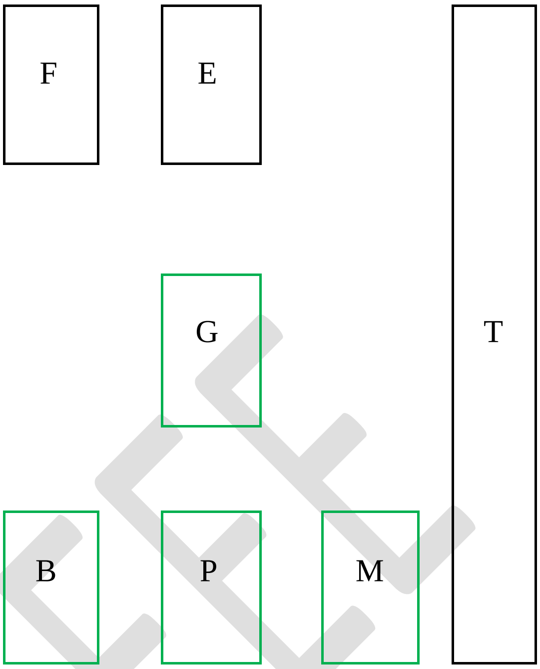


Figure 1b: Mode 2, light load

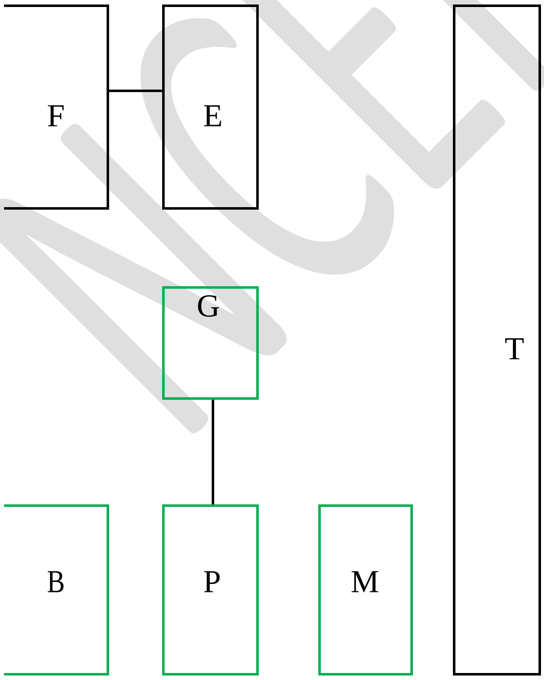


Figure 1c: Mode 3, braking or deceleration [1]

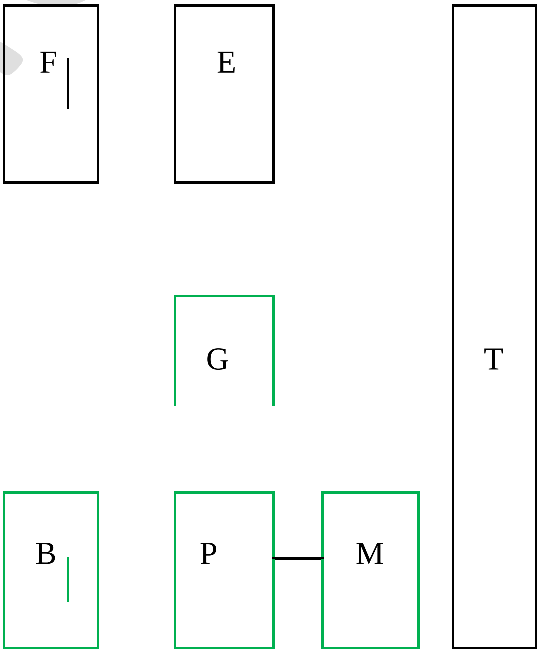


Figure 1d: Mode 4, vehicle at stop

B:Battery G: Generator —— Electrical link

E: ICE M: Motor — Hydraulic link

F: Fuel tank P: Power Converter Mechanical link

T: Transmission (including brakes, clutches and gears)

Power Flow Control in Parallel Hybrid

The parallel hybrid system has four modes of operation. These four modes of operation are

- **Mode 1:** During start up or full throttle acceleration (**Figure 2a)**; both the ICE and the EM share the required power to propel the vehicle. Typically, the relative distribution between the ICE and electric motor is 80-20%.
- **Mode 2:** During normal driving (**Figure 2b**), the required traction power is supplied by the ICE only and the EM remains in off mode.
- **Mode 3:** During braking or deceleration (**Figure 2c**), the EM acts as a generator to charge the battery via the power converter.
- **Mode 4:** Under light load condition (**Figure 2d**), the traction power is delivered by the ICE and the ICE also charges the battery via the EM.

F E

T

B P M

Figure 2a: Mode 1, start up

F E

T

B P M

Figure 2b: Mode 2, normal driving

F E

T

B P M

Figure 2c: Mode 3, braking or deceleration [1]

F E

T

B P M

Figure 2d: Mode 4, light load

Electrical link

Hydraulic link

Mechanical link

B:Battery G: Generator

E: ICE M: Motor

F: Fuel tank P: Power Converter

T: Transmission (including brakes, clutches and gears)

Power Flow Control Series-Parallel Hybrid

The series-parallel hybrid system involves the features of series and parallel hybrid systems. Hence, a number of operation modes are feasible. Therefore, these hybrid systems are classified into two categories: **the ICE dominated** and the **EM dominated**.

The various operating modes of **ICE dominated** system are:

- **Mode 1:** At startup (**Figure 3a)**, the battery solely provides the necessary power to propel the vehicle and the ICE remains in off mode.
- **Mode 2:** During full throttle acceleration (**Figure 3b**), both the ICE and the EM share the required traction power.
- **Mode 3:** During normal driving (**Figure 3c**), the required traction power is provided by the ICE only and the EM remains in the off state.
- **Mode 4:** During normal braking or deceleration (**Figure 3d**), the EM acts as a generator to charge the battery.

- **Mode 5:** To charge the battery during driving (**Figure 3e**), the ICE delivers the required traction power and also charges the battery. In this mode the EM acts as a generator.

- **Mode 6:** When the vehicle is at standstill (**Figure 3f**), the ICE can deliver power to charge the battery via the EM

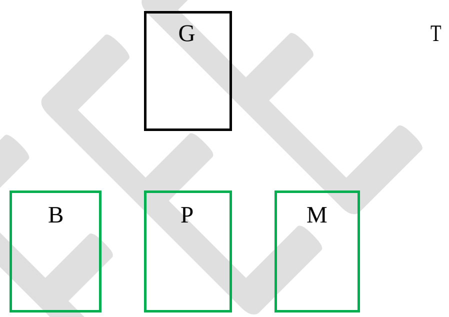


Figure 3e: Mode 5, battery charging during driving [1]

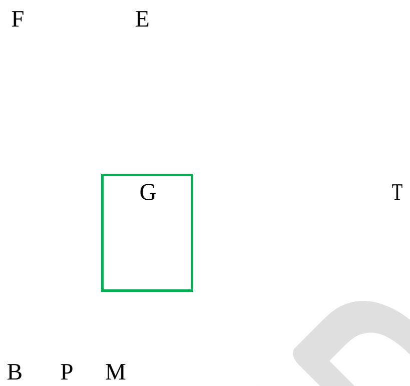


Figure 3a: Mode 1, start up [1]

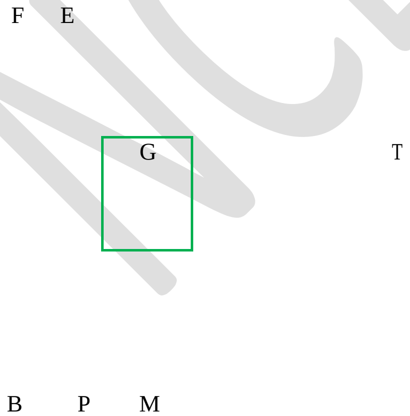


Figure 3c: Mode 3, normal drive [1]

F E

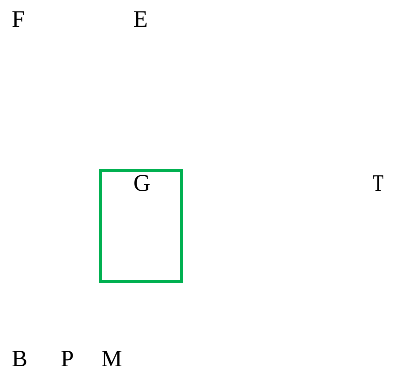
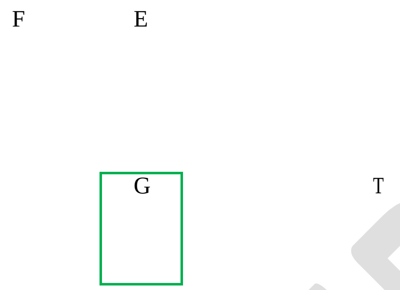


Figure 3b: Mode 2, acceleration [1]



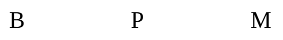


Figure 3d: Mode 4, braking or deceleration [1]

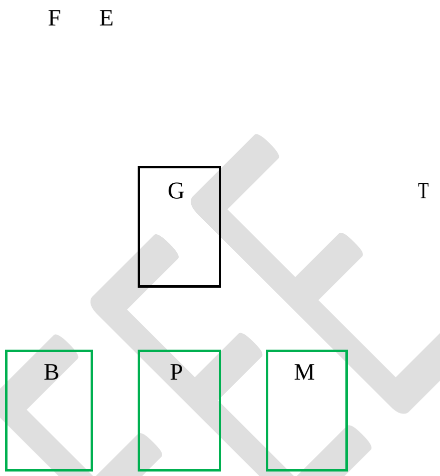


Figure 3f: Mode 6, battery charging during standstill

[1]

The operating modes of **EM dominated** system are:

- **Mode 1:** During startup (**Figure 4a**), the EM provides the traction power and the ICE remains in the off state.
- **Mode 2:** During full throttle (**Figure 4b**), both the ICE and EM provide the traction power.
- **Mode 3:** During normal driving (**Figure 4c**), both the ICE and EM provide the traction power.
- **Mode 4:** During braking or deceleration (**Figure 4d**), the EM acts as a generator to charge the battery.
- **Mode 5:** To charge the battery during driving (**Figure 4e**), the ICE delivers the required traction power and also charges the battery. The EM acts as a generator.
- **Mode 6:** When the vehicle is at standstill (**Figure 4f**), the ICE can deliver power to charge the battery via the EM

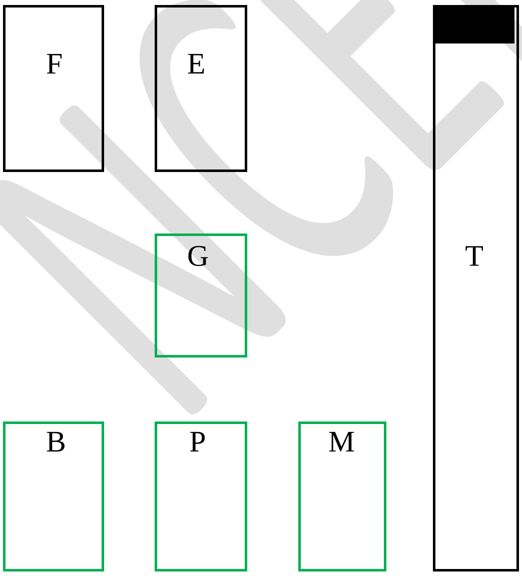


Figure 4a: Mode 1, start up [1]

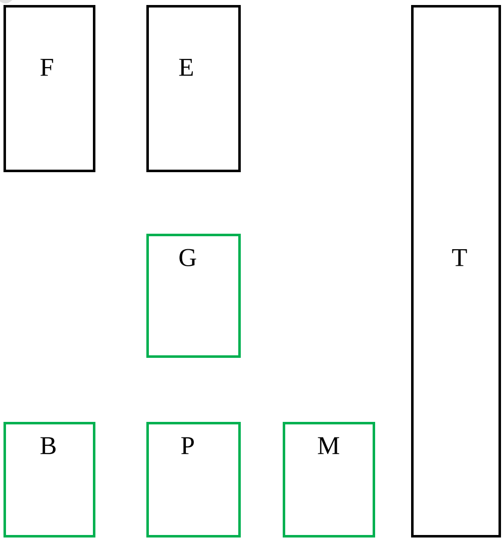
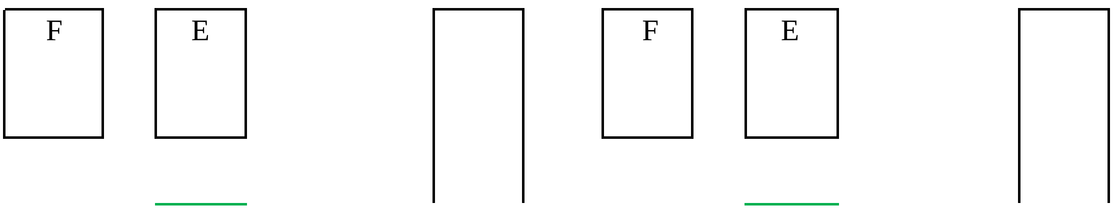


Figure 4b: Mode 2, acceleration [1]



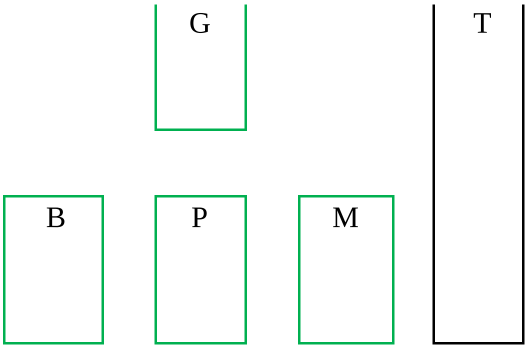


Figure4c: Mode 3, normal drive [1]

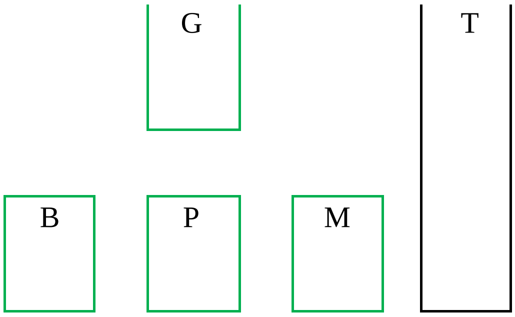


Figure 4d: Mode 4, braking or deceleration [1]

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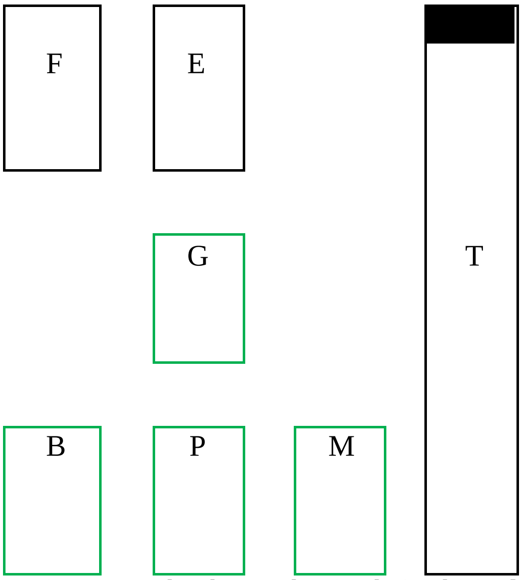


Figure 4e: Mode 5, battery charging during driving [1]

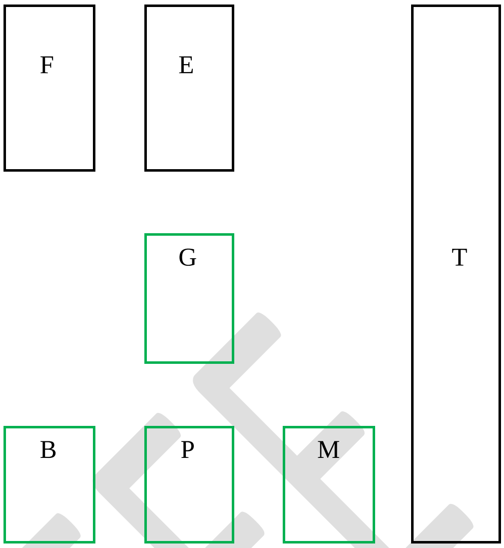


Figure 4f: Mode 6, battery charging during standstill [1]

Power Flow Control Complex Hybrid Control

The complex hybrid vehicle configurations are of two types:

ιϖ. Front hybrid rear electric

ϖ. Front electric and rear hybrid

- **Mode 1:** During startup (**Figure 5a**), the required traction power is delivered by the EMs and the engine is in off mode.

- **Mode 2:** During full throttle acceleration (**Figure 5b**), both the ICE and the front wheel EM deliver the power to the front wheel and the second EM delivers power to the rear wheel.

- **Mode 3:** During normal driving (**Figure 5c**), the ICE delivers power to propel the front wheel and to drive the first EM as a generator to charge the battery.

- **Mode 4:** During driving at light load (**Figure 5d**) first EM delivers the required traction power to the front wheel. The second EM and the ICE are in off sate.

- **Mode 5:** During braking or deceleration (**Figure 5e**), both the front and rear wheel EMs act as generators to simultaneously charge the battery.

Ω **Mode 6:** A unique operating mode of complex hybrid system is **axial balancing**. In this mode (**Figure 5f**) if the front wheel slips, the front EM works as a generator to absorb the change of ICE power. Through the battery, this power difference is then used to drive the rear wheels to achieve the axle balancing.

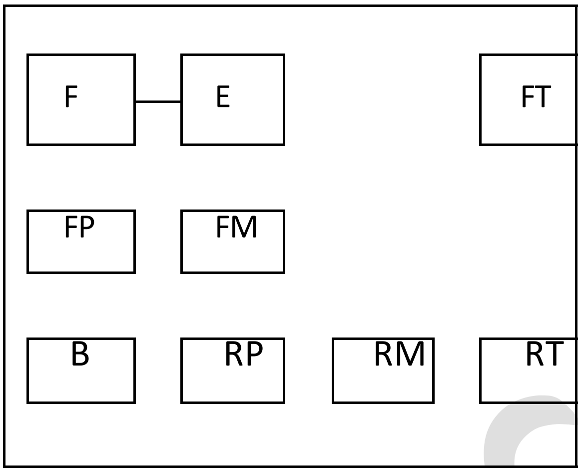


Figure 5a: Mode 1, startup

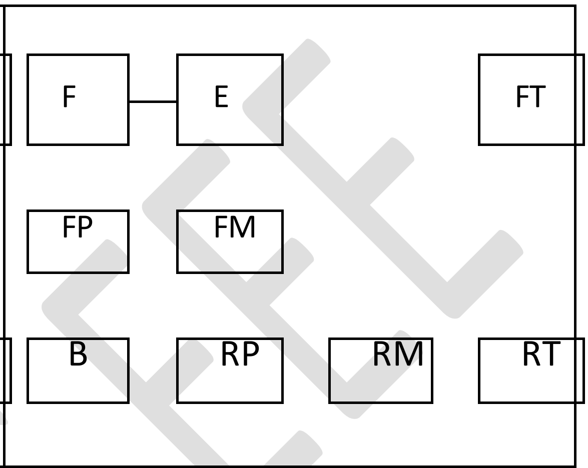


Figure 5b: Mode 2, full throttle acceleration

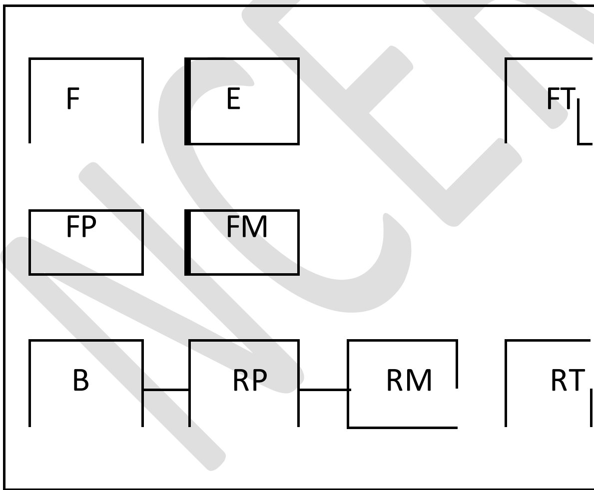


Figure 5c: Mode 3, vehicle propel and battery charging

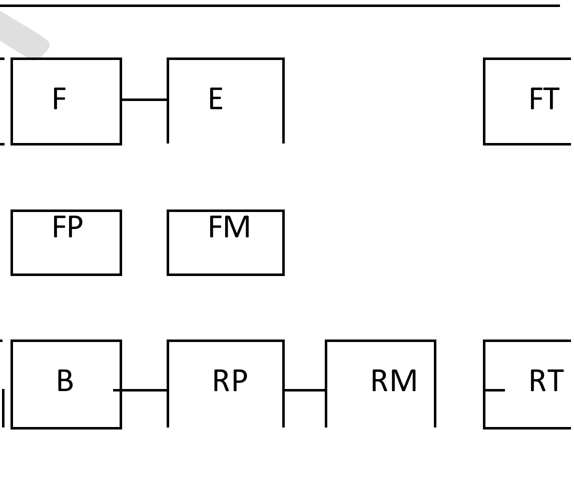
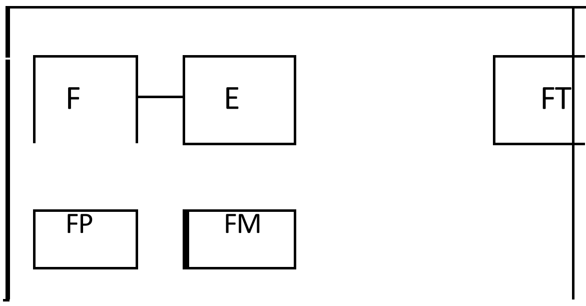
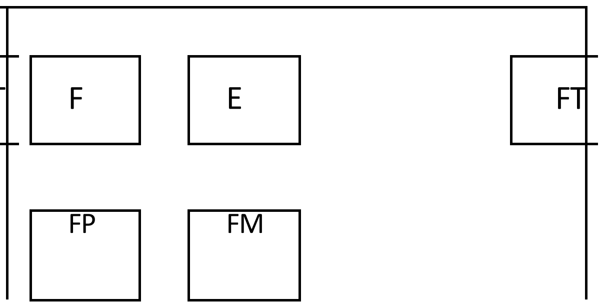


Figure 5d: Mode 4, light load





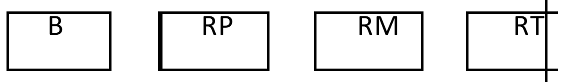


Figure 5e: Mode 5, braking or decelration

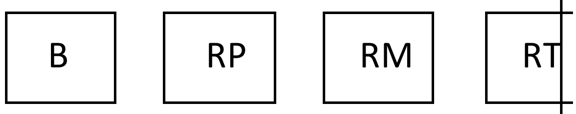


Figure 5f: Mode 1, axil balancing

B:Battery FM: Front motor FP: Front power converter FT: Front axel transmission E: ICE F: Fuel tank

RM: Rear motor RP: Rear power converter RT: Rear axle transmission [1]

Electrical link

Hydraulic link

Mechanical link

In **Figures 6a-f** all the six modes of operation of front electric and rear hybrid is shown.

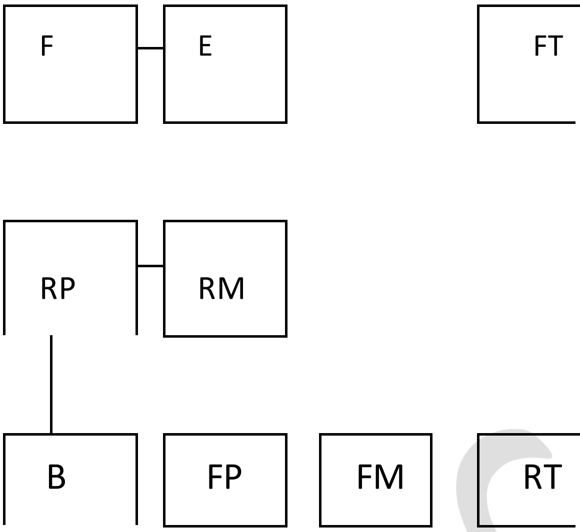


Figure 6a: Mode 1, startup

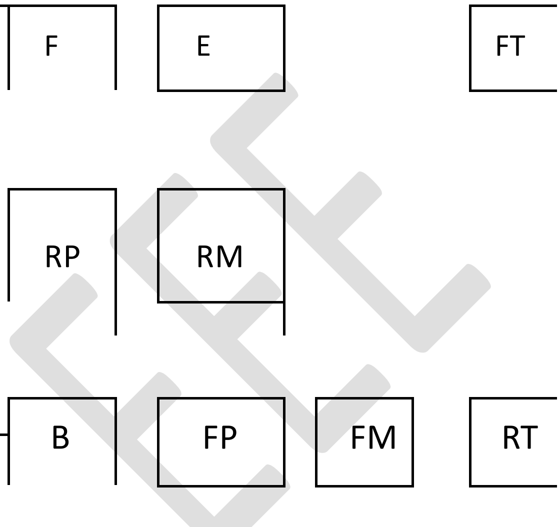


Figure 6b: Mode 2, full throttle acceleration

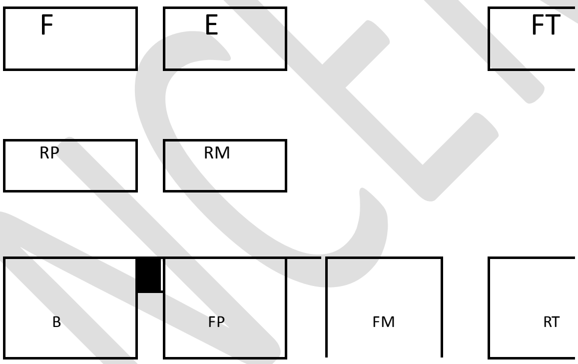


Figure 6c: Mode 3, vehicle propel and battery charging

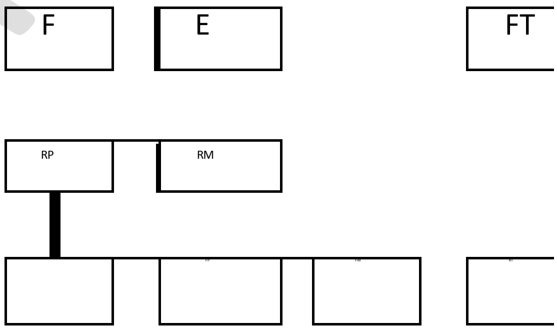
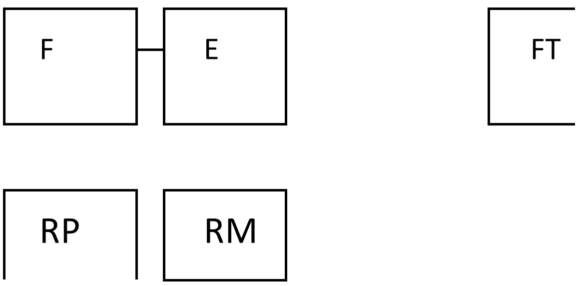
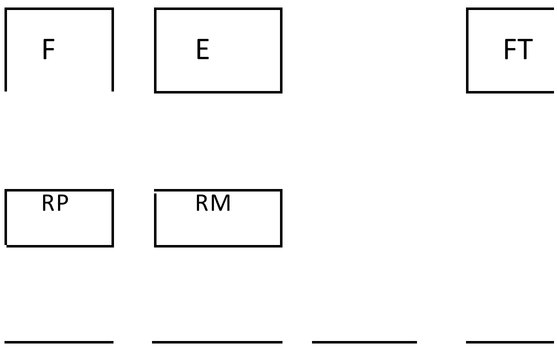
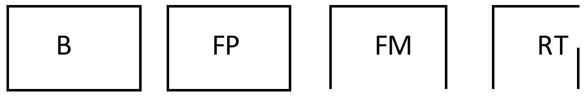


Figure 6d: Mode 4, light load







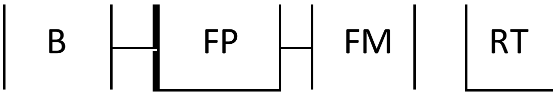


Figure 6e: Mode 5, braking or decelration

Figure 6f: Mode 1, axil balancing

B:Battery RM: Rear motor FP: Front power converter FT: Front axle transmission E: ICE F: Fuel tank

RM: Rear motor RP: Rear power converter RT: Rear axle transmission [1]

Electrical link

Hydraulic link

Mechanical link

References:

- I. Husain, *Electric and Hybrid Electric Vehicles*, CRC Press, 2003

Suggested Reading:

- M. Ehsani, *Modern Electric, Hybrid Electric and Fuel Cell Vehicles: Fundamentals, Theory and Design*, CRC Press, 2005

Lecture 7: Torque Coupling and Analysis of Parallel Drive Train

Torque Coupling and Analysis of Parallel Drive Train

Introduction

The topics covered in this chapter are as follows:

[3] Introduction to Parallel Hybrid Electric Drive Train

[4] Torque Coupling

[5] Speed Coupling

[6] Post-Transmission Parallel Hybrid Drive Train with Torque Coupling

[7] Pre-Transmission Parallel Hybrid Drive Train with Torque Coupling

[8] Parallel Hybrid Drive Train with Speed Coupling

[9] Complex Hybrid Drivetrain

Parallel Hybrid Electric Drive Trains

In case of parallel hybrid drivetrains, the ICE and an electric motor (EM) supply the required traction power. The power from ICE and EM are added together by a mechanical coupler, **Figure 1**. Generally, the mechanical coupling is of two types:

- **Torque coupling:** In this case the coupler adds the torques of the ICE and EM together and delivers the total torque to the driven wheels. The ICE and EM torque can be independently controlled. The speeds of the ICE, EM and the vehicle are linked together with a fixed relationship and cannot be independently controlled because of the power conservation constraint.

- **Speed coupling:** In this case the speeds of the ICE and EM can be added together and all torques are linked together and cannot be independently controlled.

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Fuel tank

Final drive

and differential

Engine

Mechanical

Transmission

Battery

pack

Motor

controller

Battery

charger

Traction

Battery charge

Figure 1:General Configuration of a Parallel Hybrid Drive Train [1]

Torque Coupling

In **Figure 2**, a conceptual diagram of mechanical torque coupling is shown. The torque coupling, shown in **Figure 2**, is a two-degree-of-freedom mechanical device. Port 1 is a unidirectional input and Port 2 and 3 are bi-directional input or output, but both are not input at the same time. Here input means the energy flows into the device and output means the energy flows out of the device. In case of HEV

- **port 1** is connected to the shaft of an **ICE** directly or through a mechanical transmission.
- **port 2** is connected to the shaft of an **electric motor** directly or through a mechanical transmission
- **port 3** is connected to the driven **wheels** through a mechanical linkage

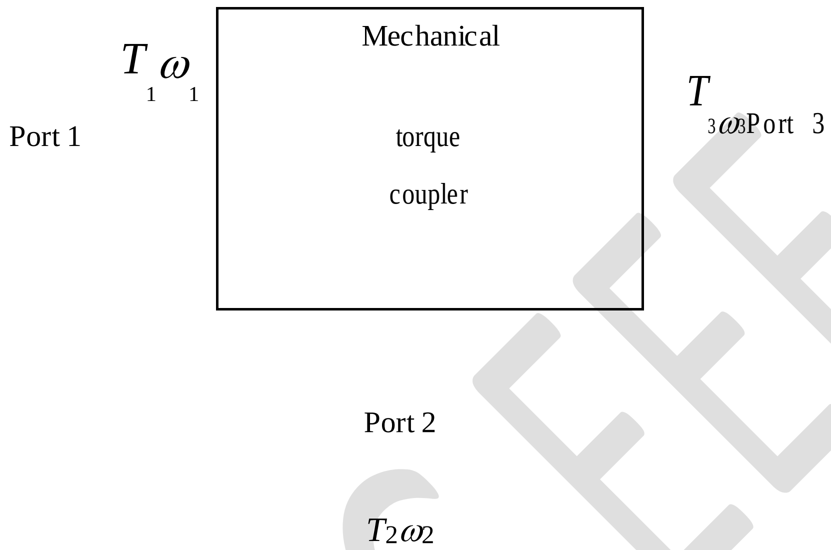

Figure 2:Mechanical torque coupler [2]

For a ***losses*** torque coupler in ***steady state***, the power input is always equal to the power output from it. For the torque coupler shown in **Figure 1**, the power balance is

$$T_3\omega_3 = T_1\omega_1 + T_2\omega_2 \quad (1)$$

where

T_1 = Propelling torque produced by ICE; ω_1 = Speed of ICE

T_2 =Propelling torque produced by EM; ω_2 = Speed of EM

T_3 = Load torque delivered to wheels; ω_3 = Speed of wheel

The torque coupler can be expressed as

$$T_3 = k_1T_1 + k_2T_2 \quad (2)$$

where

k_1, k_2 are the structural parameters of the torque coupler

From **equation 1** and **equation 2** it can be seen that

$$\omega_3 = \frac{\omega_1}{k_1} = \frac{\omega_2}{k_2} \quad (3)$$

A gearbox used in the vehicles is a typical example of torque couple. Some torque coupler are shown in **Figure 3**

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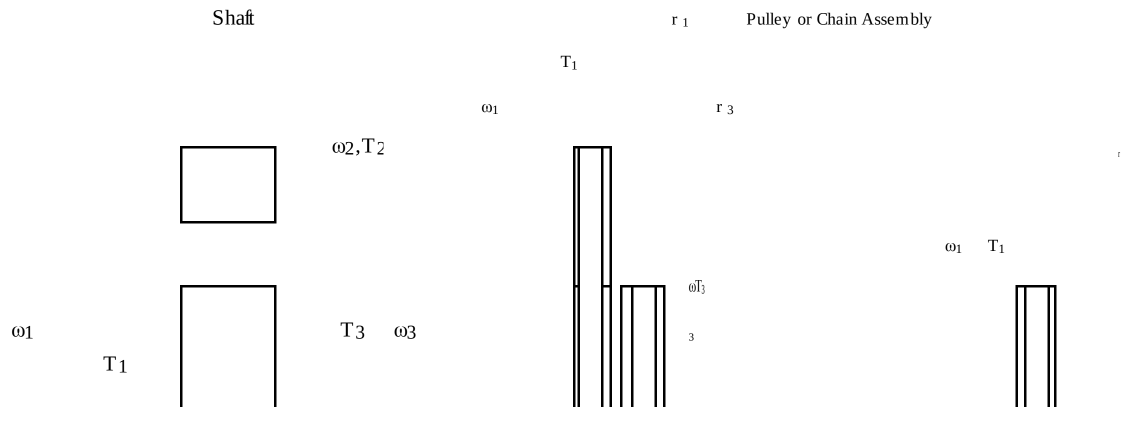

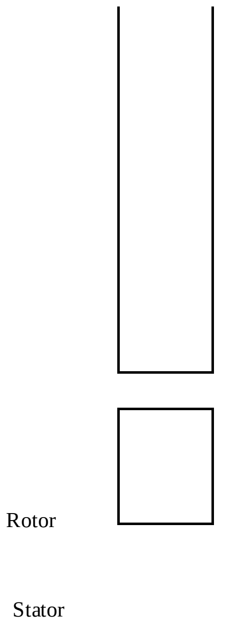

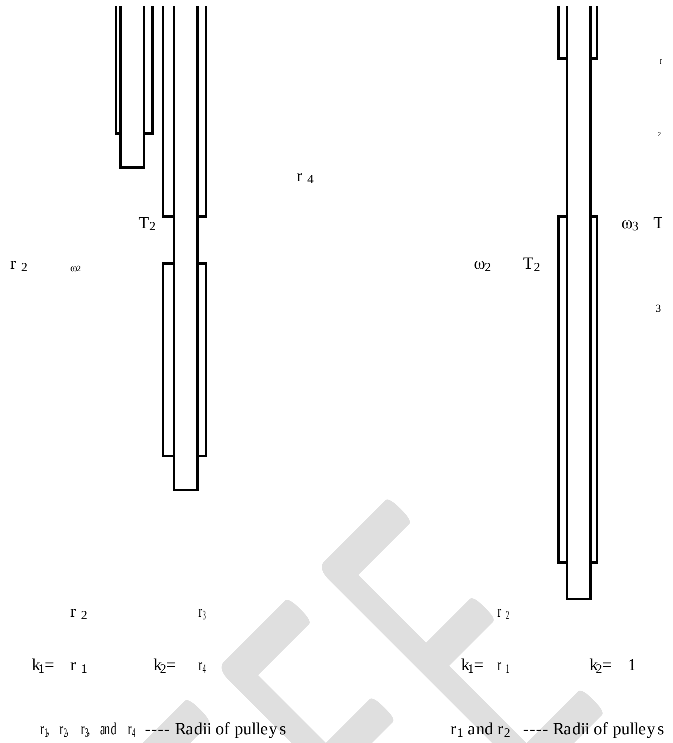

Figure 3a:Configuration of a torque coupler [2]

Figure 3b:Configuration of a pulley/chain assembly torque coupler [2]

Speed Coupling

The power produced by two power plants may be coupled together by adding their speed. This is done with the help of **speed coupling** devices (**Figure 4**). The Speed Coupler is a three port two-degree-of-freedom device. Port 1 is a unidirectional input and Port 2 and 3 are bi-directional input or output, but both are not input at the same time. Here input means the energy flows into the device and output means the energy flows out of the device. In case of HEV

- **port 1** is connected to the shaft of an **ICE** directly or through a mechanical transmission.
- **port 2** is connected to the shaft of an **electric motor** directly or through a mechanical transmission
- **port 3** is connected to the driven **wheels** through a mechanical linkage

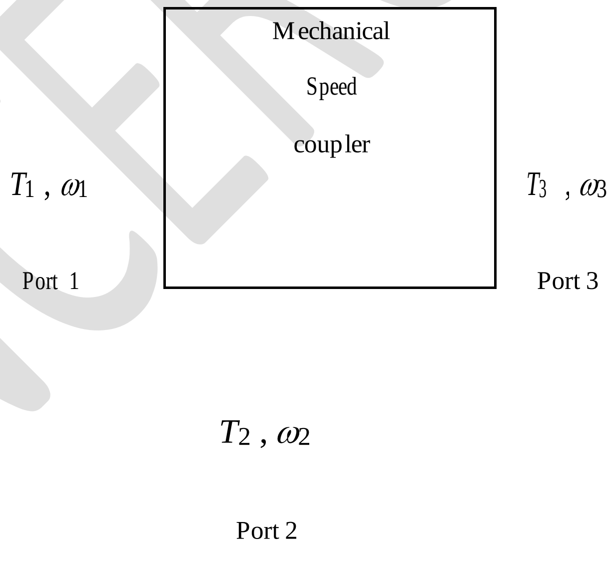

Figure 4:Mechanical speed coupler [2]

For a ***losses*** speed coupler in ***steady state***, the power input is always equal to the power output from it. For the speed coupler shown in **Figure 4**, the speed relation is

$$\omega_3 = k_1\omega_1 + k_2\omega_2$$

where (4)

k_1 , k_2 are the structural parameters of the speed coupler

The power relation in case of speed coupler is same as given in **equation 1**. From **equation 1** and **equation 4** it can be seen that

$$T_3 = \frac{T_1}{k_1} = \frac{T_2}{k_2} \quad (5)$$

A typical speed coupler is the planetary gear (**Figure 5**). The planetary gear unit is a three port device consisting of

- Sun gear, marked **1** in **Figure 5**
- Ring gear, marked **2** in **Figure 5**
- Carrier or Yoke, marked **3** in **Figure 5**

T_2, ω_2

2

3

3 2

r_c

r_s r_r

1 T_1, ω_1

ω_3 T_3

1

Figure 5a:Planetary gear front view [2] **Figure 5b:Planetary gear cut section [2]**

For a planetary gear train configuration as shown in **Figure 5**, the gear ratio (n_b) is given by

$$n_b = \frac{\omega_s - \omega_c}{\omega_r - \omega_c} = -\frac{z_r}{z_c}\frac{z_c}{z_s} = -\frac{z_r}{z_s} = -\frac{r_r}{r_s}$$

where

ω_s = angular speed of the sun gear

ω_c = angular speed of the carrier gear

ω_r = angular speed of the ring gear

(6)

z_s = number of teeth on the sun gear

z_r = number of teeth on the ring gear

z_c = number of teeth on the carrier gear

r_s = radius of the sun gear

r_r = radius of the ring gear

r_c = radius of the carrier gear

The **equation 6** can also be expressed as

$$n_b\left(\omega_r - \omega_c\right) = \omega_s - \omega_c$$

$$\Rightarrow \omega_s - n_b\omega_r - \omega_c\left(1 - n_b\right) = 0 \qquad (7)$$

$$\Rightarrow \omega_c = \frac{1}{1 - n_b}\omega_s - \frac{n_b}{1 - n_b}\omega_r$$

In the analysis of the planetary gears, rotation and torque in the anticlockwise direction is assumed to be positive and in the clockwise direction is assumed to be negative. Using the power balance, the torque acting on each gear is obtained as

$$T_s\omega_s + T_c\omega_c + T_r\omega_r = 0 \qquad (8)$$

Substituting the value of ω_c from **equation 7** into **equation 8** gives

$$T_s\omega_s + T_c\left(\frac{\omega_s}{1 - n_b} - \frac{n_b}{1 - n_b}\omega_r\right) + T_r\omega_r = 0$$

$$\Rightarrow \omega_s\left(T_s + \frac{1}{1 - n_b}T_c\right) + \omega_r\left(T_r - \frac{n_b}{1 - n_b}T_c\right) = 0 \qquad (9)$$

$$T_c = -\left(1 - n_b\right)T_s \text{ and } T_c = \frac{1 - n_b}{n_b}T_r$$

If the carrier is attached to a stationary frame ($\omega_c = 0$) then from **equation 7**

$$n_b = \frac{\omega_s}{\omega_r} = -\frac{z_r}{z_s} = -\frac{r_r}{r_s} \qquad (10)$$

and from **equation 8** the torque relation is given by

$$T_s = -\frac{\omega_r}{\omega_s} T_r \qquad (11)$$

$$\Rightarrow T_s = -\frac{T_r}{n_b}$$

From **Figure 5** it can be seen that $r_r > r_s$, hence $n_b > 1$. If it is assumed that the input torque is given to the sun gear and the output shaft is connected to the ring gear, then from **equation 10** and **equation 11** it can be deduced that

- The output torque (T_r) is increased by a factor n_b and the direction of the output torque is same as that of the input torque (T_s)
- The output speed (ω_r) is reduced by a factor of n_b and the direction of speed is reversed with respect to the input speed (ω_s).

In **Table 1** all the six possible scenarios of planetary gears are summarized.

Table 1: Planetary gear operation scenarios

Sun gear	Carrier gear	Ring gear	Output Speed	Output Torque	Output speed direction	Output speed magnitude	Output torque direction	Output torque magnitude
Input	Output	Held	$\omega_r = \frac{\omega_s}{n_b}$	$T_r = -n_b T_s$	Reverses	Decreases	Remains same	Increases
Held	Output	Input	$\omega_c = -\frac{n_b}{1-n_b}\omega_r$	$T_c = \frac{1-n_b}{n_b} T_r$	Remains same	Decreases	Reverses	Increases
Output	Input	Held	$\omega_s = (1-n_b)\omega_c$	$T_s = -\frac{T_c}{(1-n_b)}$	Remains same	Increases	Reverses	Decreases
Held	Input	Output	$\omega_r = -\frac{1-n_b}{n_b}\omega_c$	$T_r = \frac{n_b}{1-n_b} T_c$	Remains same	Increases	Reverses	Decreases
Input	Held	Output	$\omega_r = \frac{\omega_s}{n_b}$	$T_r = -n_b T_s$	Reverses	Decreases	Remains same	Increases
Output	Held	Input	$\omega_s = n_b \omega_r$	$T_s = -\frac{T_r}{n_b}$	Reverses	Increases	Remains same	Decreases

Parallel Hybrid Drive Train with Torque Coupling (Post-transmission)

In **Figure 6** a two-shaft configuration of parallel HEV using torque coupler is shown.

In this case two transmissions are used:

- Transmission 1 is between the ICE and the torque coupler
- Transmission 2 is between the EM and the torque coupler

Both the transmissions (***Transmission 1*** and ***Transmission 2***) may be single geared or multigear. The possible configurations are:

- **Configuration 1:** Both, ***Transmission 1*** and ***Transmission 2*** are multigear. The tractive effort vs. speed profile is shown in **Figure 7a**.
- **Configuration 2:** ***Transmission 1*** is multigear and ***Transmission 2*** is single geared (**Figure 7b**)
- **Configuration 3:** ***Transmission 1*** is single geared and ***Transmission 2*** is multigear (**Figure 7c**)

- **Configuration 4:** Both, ***Transmission 1*** and ***Transmission 2*** are single geared (**Figure 7d**)

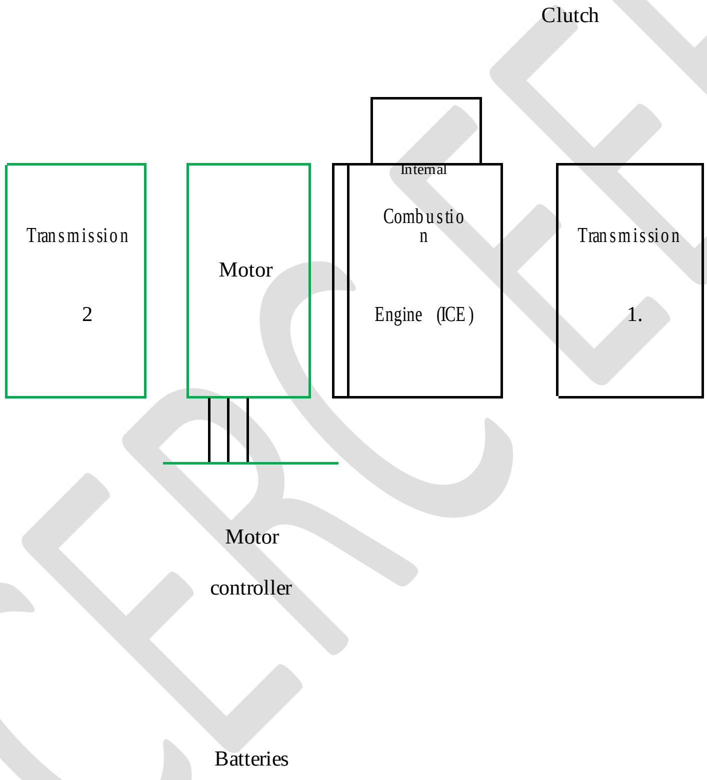

Figure 6:Dual transmission parallel hybrid drivetrain [3]

1 2nd ge

Transmission 2 1st gea

Transmission 2 2nd ge

Transmission 1 1st gea

Transmission 1 2nd ge

Transmission 2 1st gea

Transmission 2 3rd ge

Transmission 1 1st gea

Transmission 1 3rd ge

Transmission 2 2nd ge

Transmission 2 1st gea

Transmission 1 1st gea

Transmission 1 3rd ge

Transmission 2 3rd ge

Transmission 2 2nd ge

Transmission 1 2nd ge

Transmission

3rd gear

Transmission 1 3rd ge

Transmission 2 1st gear

Transmission 2 3rd ge

Vehicle Speed [km/h]

Vehicle Speed [km/h]

Transmission 1 1st gear

Transmission 2 1st gear

Transmission 1 2nd gear

Transmission 2 1st gear

Transmission 1

Figure 7a: Transmission 1 and Transmission 2 are multigeared [3]

Figure 7b: Transmission 1 is multigear and Transmission 2 is single geared [3]

Transmission 1 1st gear

Transmission 2 1st gear

effort [N]

Transmission 1 1st gear

Transmission 2 2nd gear

Transmissi on 1 1st ge

Transmissi on 2 1st ge

Transmission 1 1st gear

Transmission 2 3rd gear

Vehicle Speed [km/h]

Vehicle Speed [km/h]

Figure 7c: Transmission 1 is single geared and Transmission 2 is multigear [3]

Figure 7d: Transmission 1 and Transmission 2 are single geared [3]

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Upon analyzing the tractive effort vs. speed profile of **Configuration 1** it can be concluded that:

- Two multigear transmissions produce many tractive effort profiles. Hence, the performance and overall efficiency of the drive train may be superior to other designs because two multigear transmissions provide more opportunities for both the ICE and the EM-drive (electric motor and the associated power electronics) to operate in their optimum region.
- This configuration provides more opportunities for both the ICE and EM characteristics.
- The control system for selecting the proper gear in each transmission is complicated.

- The multigear ***Transmission 1*** is used to overcome the disadvantage of the ICE speed vs. torque characteristics.
- The multigear ***Transmission 1*** also improves the operating efficiency of the engine and reduces the speed range of the vehicle in which EM must be used to propel the vehicle. Hence, the use of EM is restricted and this prevents the batteries from quickly discharging.
- The single gear ***Transmission 2*** takes the advantage of the high torque of an EM at low speed.

The **Configuration 3** is unfavorable because it does not use the advantages of the two power plants. The **Configuration 4** results in a simple design and control. With proper ratings of the ICE, EM, batteries and transmission parameters, this drivetrain can serve the vehicle with satisfactory performance and efficiency.

Parallel Hybrid Drive Train with Torque Coupling (Pre-transmission)

The pre-transmission configuration of a parallel HEV with torque coupling is shown in **Figure 8**. In this configuration the transmission is located between the torque coupler and the drive shaft. The transmission amplifies the torques of both the ICE and the EM with the same scale. The design of the gear ratios in the torque coupler enables the EM and ICE to reach their maximum speeds at the same time. This configuration is suitable when relatively small EM and ICE are used, where a multigear transmission is needed to enhance the tractive effort at low speeds.

Battery charger

Motor controller

Engine

Transmission

V

T_{ice} T_m

ω_{ice} ω_m

Figure 8: Pre transmission parallel hybrid drive [3]

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Parallel Hybrid Drive Train with Speed Coupling

In **Figure 9** a parallel hybrid drive train with speed coupling using planetary gear unit and an EM. The connection of the ICE and the EM is as follows:

- The engine supplies its power to the sun gear through a clutch and transmission. The transmission is used to modify the speed vs. torque profile of the ICE so as to match the traction requirements. The transmission may be single gear or multigear.
- The EM supplies its power to the ring gear through a pair of gears. The Locks 1 and 2 are used to lock the sun gear and ring gear to the stationary frame of the vehicle in order to implement different modes of operation.

Lock2

Clutch

Lock1

Engine

Trans.

Motor

Motor

Batteries

controller

Figure 9: Parallel hybrid drive with speed coupling [1]

There are 5 different modes of operation possible for the configuration as shown in

Figure 10 and they are:

- **Hybrid traction:** When lock 1 and lock 2 are released, i.e. the sun gear and the ring gear can rotate both the ICE and EM supply positive speed and torque to the driven wheels. Since, the output shaft is connected to the carrier gear, the output torque and speed is give by

$$T_c = (n_b - 1)\frac{T_s\omega_s + T_r\omega_r}{\omega_r n_b - \omega_s} \qquad (12)$$

$$\omega_c = \frac{1}{1-n_b}\omega_s - \frac{n_b}{1-n_b}\omega_r \qquad (13)$$

- **Engine alone traction**: When the lock 2 locks the ring gear, only the ICE delivers the required traction force to the wheels. The output torque and the speed is given by

$$T_c = (1 - n_b) T_s \tag{14}$$

$$\omega_c = \frac{1}{1 - n_b} \omega_s \tag{15}$$

- **Motor alone traction:** When lock 1 locks the sun gear, only the EM delivers the traction force to the wheels. The output torque and the speed is given by

$$T_c = \frac{1 - n_b}{n_b} T_r \tag{16}$$

$$\omega_c = -\frac{n_b}{1 - n_b} \omega_r \tag{17}$$

- **Regenerative braking:** In this case lock 2 is engaged, the ICE is switched off, the clutch is disengaged and the EM is controlled in regenerating mode and the battery absorbs the kinetic energy of the vehicle.

- **Battery charging from the ICE:** In this mode the locks 1 and 2 are released. The EM is controlled to rotate in the opposite direction, i.e. the EM operates with positive torque and negative speed and absorbs power from the engine and delivers it to the battery.

Complex Hybrid Drive Train Drivetrain

In **Figure 10**, a complex HEV drivetrain with both torque and speed coupling is shown. This architecture is used by Toyota Prius. The main components of Prius drivetrain are

- **Planetary gear unit:** Used for speed coupling
- **Fixed Axel Gear:** Used for torque coupling

The various power sources of Prius drivetrain are connected as follows:

- The ICE is connected to the carrier gear of the planetary
- A small EM (EM1) is connected to the sun gear
- The ring gear is connected to the driven wheels through axel fixed gear unit (torque coupler)
- An EM (EM2) is also connected to the fixed angle axel gear unit and forms the torque coupling configuration.

The rotational speed of the ring gear is given by

$$\omega_r = \frac{1}{n_b}\omega_s - \frac{1-n_b}{n_b}\omega_c \qquad (18)$$

Since the ICE is connected to the carrier gear and the EM1 is connected to the sun gear, the **equation 18** can be expressed as

$$\omega_r = \frac{1}{n_b}\omega_{ICE} - \frac{1-n_b}{n_b}\omega_{EM1} \qquad (19)$$

where

ω_{ICE} = angular speed of the ICE

ω_{EM1} = angular speed of the EM1

The various modes of operation are:

- **Mode 1:** When the vehicle speed is low and the ICE speed is not so low then EM1 rotates in the positive direction (same direction as ICE). In this condition, the EM1 operates in generation mode and a fraction of ICE power is used to charge the battery.
- **Mode 2:** At higher vehicle speed, while trying to maintain the engine speed below a given speed, for high engine operating efficiency, the EM1 may be operated in negative speed. In this case EM1 acts as a motor and delivers power to propel the vehicle.

The traction motor EM2 adds additional torque to the torque output from the ring gear of the planetary gear unit using torque coupling device.

Clutch

Lock2

Lock1

Z_r

Clutch3 Z_d Z_p

Z_b

Z_s

Engine

Trans.

Motor

Z_a Z_c

Batteries

Motor

controller

Clutch2

Figure 10: Complex hybrid drive with speed and torque coupling [1]

References

[1] M. Ehsani, *Modern Electric, Hybrid Electric and Fuel Cell Vehicles: Fundamentals, Theory and Design*, CRC Press, 2005

[2] L. Guzzella and A. Sciarretta, *Vehicle Propulsion Systems: Introduction to Modeling and Optimization*, Springer, 2007

[3] G. Lechner and H. Naunheimer, *Automotive Transmissions: Fundamentals, Selection, Design and Application*, Springer, 1999

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Lecture 8: Basic Architecture of Electric Drive Trains

Basic Architecture of Electric Drive Trains

Introduction

The topics covered in this chapter are as follows:

- Electric Vehicle (EV) Configuration
- EV alternatives based on drivetrains
- EV alternatives based on power source configuration
- Single and Multi-motor drives
- In wheel drives

Electric Vehicle (EV) Configurations

Compared to HEV, the configuration of EV is flexible. The reasons for this flexibility are:

- The energy flow in EV is mainly via flexible electrical wires rather than bolted flanges or rigid shafts. Hence, distributed subsystems in the EV are really achievable.
- The EVs allow different propulsion arrangements such as independent four wheels and in wheel drives.

In **Figure 1** the general configuration of the EV is shown. The EV has three major subsystems:

- Electric propulsion
- Energy source

- Auxiliary system

- The electronic controller
- Power converter
- Electric Motor (EM)
- Mechanical transmission
- Driving wheels

The energy source subsystem consists of

- The energy source (battery, fuel cell, ultracapacitor)
- Energy management unit
- Energy refueling unit

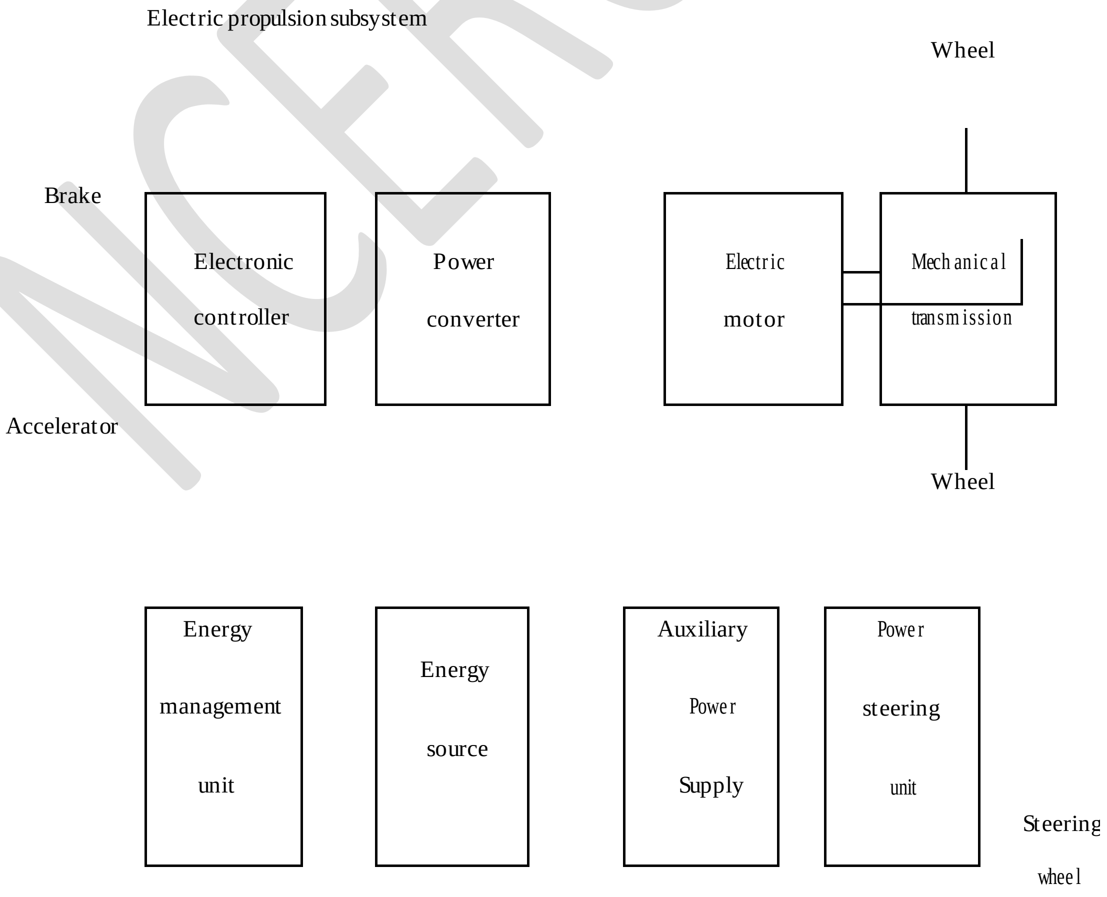
- Power steering unit
- Temperature control unit
- Auxiliary power supply

.

In **Figure 1** the black line represents the mechanical link, the green line represents the electrical link and the blue line represents the control information communication. Based on the control inputs from the brake and accelerator pedals, the electronic controller provides proper control signals to switch on or off the power converter which in turn regulates the power flow between the electric motor and the energy source. The backward power flow is due to regenerative braking of the EV and this regenerative energy can be stored provided the energy source is receptive.

The energy management unit cooperates with the electronic controller to control regenerative braking and its energy recovery. It also works with the energy-refueling unit to control refueling and to monitor usability of the energy source.

The auxiliary power supply provides the necessary power with different voltage levels for all EV auxiliaries, especially the temperature control and power steering units.



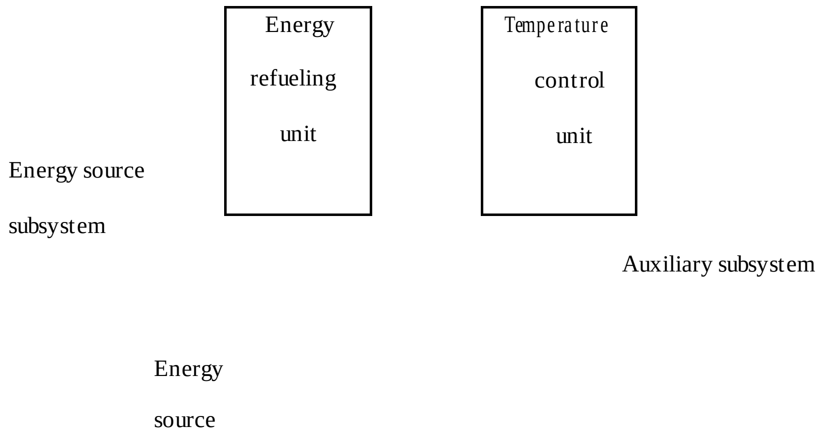
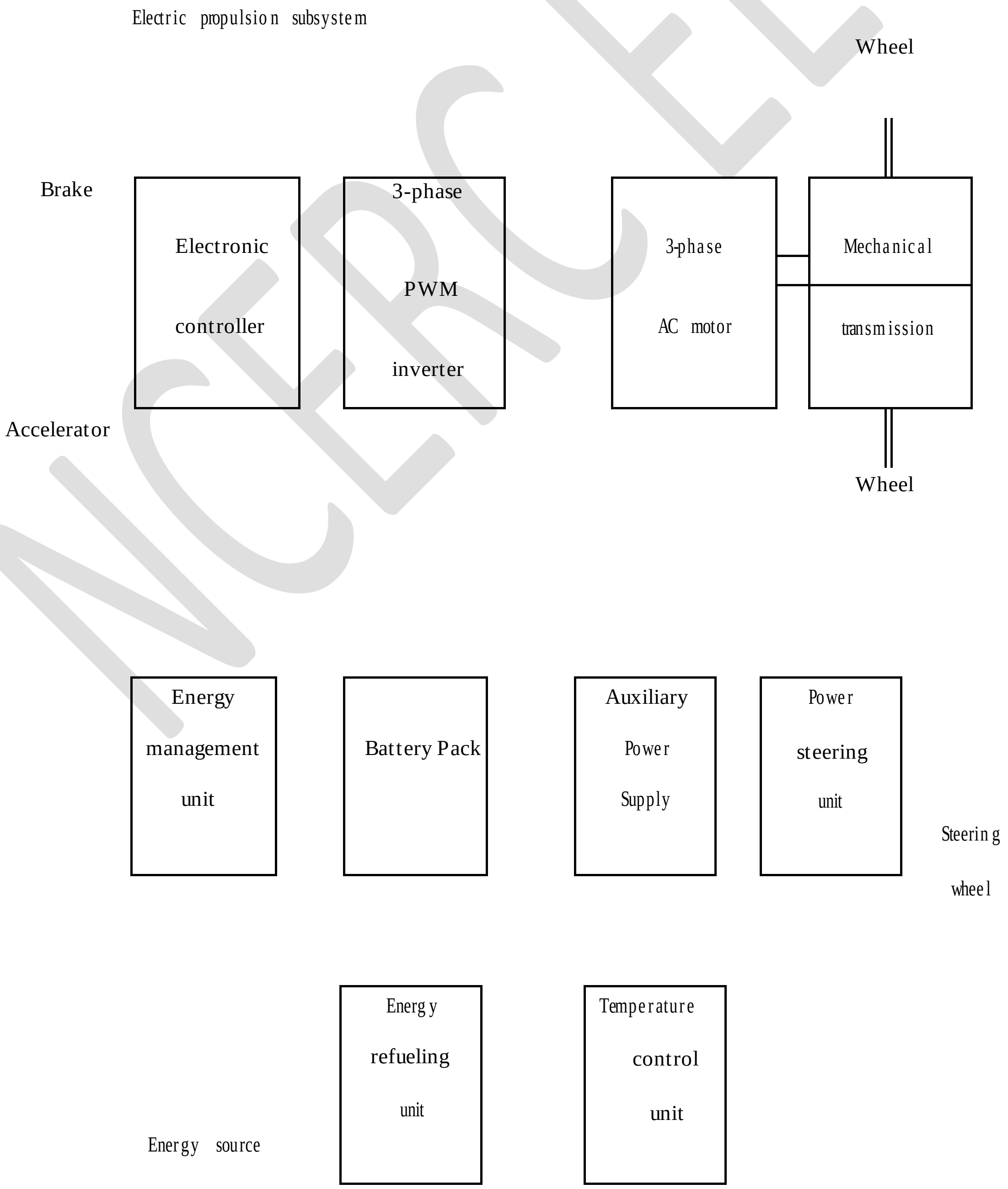


Figure 1:General Configuration of a Electric Vehicle [1]

In modern EV's configuration:

- Three phase motors are generally used to provide the traction force
- The power converter is a three-phase PWM inverter
- Mechanical transmission is based on fixed gearing and a differential
- Li-ion battery is typically selected as the energy source

The typical setup of the EV is shown in **Figure 2**.



subsystem

Auxiliary subsystem

Figure 2:Typical Configuration of a Electric Vehicle [1]

Electric Vehicle (EV) Drivetrain Alternatives Based on Drivetrain Configuration

There are many possible EV configurations due the variations in electric propulsion and energy sources. Based on these variations, six alternatives are possible as shown in **Figure 3**. These six alternatives are

- In **Figure 3a** a single EM configuration with gearbox (GB) and a clutch is shown. It consists of an EM, a clutch (C), a gearbox, and a differential (D). The clutch enables the connection or disconnection of power flow from EM to the wheels. The gear consists of a set of gears with different gear ratios. With the use of clutch and gearbox, the driver can shift the gear ratios and hence the torque going to the wheels can be changed. The wheels have high torque low speed in the lower gears and high-speed low torque in the higher gears.

- In **Figure 3b** a single EM configuration without the gearbox and the clutch is shown. The advantage of this configuration is that the weight of the transmission is reduced. However, this configuration demands a more complex control of the EM to provide the necessary torque to the wheels.

- **Figure 3c** shows a configuration of EV using one EM. It is a transverse front EM front wheel drive configuration. It has a fixed gearing and differential and they are integrated into a single assembly.

- In **Figure 3d** a dual motor configuration is shown. In this configuration the differential action of an EV when cornering can be electronically provided by two electric motors.

- In order to shorten the mechanical transmission path from the EM to the driving wheel, the EM can be placed inside a wheel. This configuration is called in-wheel drive. **Figure 3e** shows this configuration in which fixed planetary gearing is employed to reduce the motor speed to the desired wheel speed.

- In **Figure 3f** an EV configuration without any mechanical gearing is shown. By fully abandoning any mechanical gearing, the in-wheel drive can be realized by installing a low speed outer-rotor electric motor inside a wheel.

C

M GB D

MFG D

Figure 3a: EV configuration with clutch, gearbox and differential [1]

Figure 3b: EV configuration without clutch and gearbox [1]

FG

M M

FG

D

M

FG

Figure 3c:EV configuration with clutch, gearbox and differential [1]

Figure 3d:EV configuration with two EM [1]

.

FG

M

M

FG

Figure 3e:EV configuration with in wheel motor and mechanical gear [1]

Figure 3f:EV configuration with in wheel motor and no mechanical gear [1]

Electric Vehicle (EV) Drivetrain Alternatives Based on Power Source Configuration

Besides the variations in electric propulsion, there are other EV configurations due to variations in energy sources. There are five configurations possible and they are:

- **Configuration 1:** It is a simple battery powered configuration, **Figure 4a**. The battery may be distributed around the vehicle, packed together at the vehicle back or located beneath the vehicle chassis. The battery in this case should have reasonable specific energy and specific power and should be able to accept regenerative energy during braking. In case of EVs, the battery should have both high specific energy and specific power because high specific power governs the driving range while the high power density governs the acceleration rate and hill climbing capability.

- **Configuration 2:** Instead of two batteries, this design uses two different batteries, **Figure 4b**. One battery is optimized for high specific energy and the other for high specific power.

- **Configuration 3:** In this arrangement fuel cell is used, **Figure 4c**. The battery is an energy storage device, whereas the fuel cell is an energy generation device. The operation principle of fuel cells is a reverse process of electrolysis. In reverse and electrolysis, hydrogen and oxygen gases combine to form electricity and water. The hydrogen gas used by the fuel cell can be stored in an on-board tank whereas oxygen gas is extracted from air. Since fuel cell can offer high specific energy but cannot accept regenerative energy, it is preferable to combine it with battery with high specific power and high-energy receptivity.

- **Configuration 4:** Rather than storing it as a compressed gas, a liquid or a metal hydride, hydrogen can be can be generated on-board using liquid fuels such as methanol, **Figure 4d.** In this case a mini reformer is installed in the EV to produce necessary hydrogen gas for the fuel cell.

- **Configuration 5:** In fuel cell and battery combination, the battery is selected to provide high specific power and high-energy receptivity. In this configuration a battery and supercapacitor combination is used as an energy source, **Figure 4e**. The battery used in this configuration is a high energy density device whereas the supercapacitor provides high specific power and energy receptivity. Usually, the supercapacitors are of relatively low voltage levels, an additional dc-dc power converter is needed to interface between the battery and capacitor terminals.

Figure 4a: EV configuration with battery source [1]

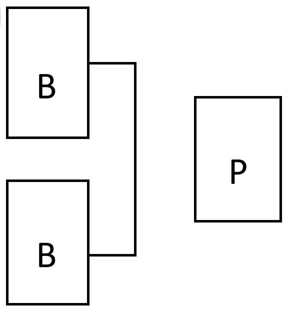
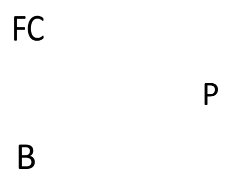


Figure 4b: EV configuration with two battery sources [1]



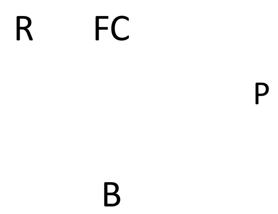


Figure 4c: EV configuration with battery and fuel cell sources [1]

Figure 4d: EV configuration with multiple energy sources [1]

B

P

C P

B

P

FW P

Figure 4e: EV configuration with battery and capacitors sources [1]

Figure 4f: EV configuration with battery and flywheel sources [1]

Single and Multi-motor Drives

A differential is a standard component for conventional vehicles. When a vehicle is rounding a curved road, the outer wheel needs to travel on a larger radius than the inner wheel. Thus, the differential adjusts the relative speeds of the wheels. If relative speeds of the wheels are not adjusted, then the wheels will slip and result in tire wear, steering difficulties and poor road holding. In case of EVs, it is possible to dispense the mechanical differential by using two or even four EMs. With the use of multiple EMs, each wheel can be coupled to an EM and this will enable independent control of speed of each wheel in such a way that the differential action can be electronically achieved. In **Figure 5**, a typical dual motor drive with an electronic differential is shown.

Wheel

Wheel

$\omega_{out} > \omega_{in}$

ω_{in}

ω_{out}

EM 1 with fixed gearing

EM 2 with fixed gearing

Figure 5: Differential action [1]

In Wheel Drives

By placing an electric motor inside the wheel, the in wheel motor has the advantage that the mechanical transmission path between the electric motor and the wheel can be minimized. Two possible configurations for in wheel drives are:

- When a high-speed inner-rotor motor is used (**Figure 6a**) then a fixed speed-reduction gear becomes necessary to attain a realistic wheel speed. In general, speed reduction is achieved using a planetary gear set. This planetary gear is mounted between the motor shaft and the wheel rim. Usually this motor is designed to operate up to 1000 rpm so as to give high power density.
- In case outer rotor motor is used (**Figure 6b**), then the transmission can be totally removed and the outer rotor acts as the wheel rim and the motor speed is equivalent to the wheel speed and no gears are required.

The tradeoffs of the high-speed inner rotor motor are:

- It has the advantage of smaller size, lighter weight and lower cost
- Needs additional planetary gearset

- Low speed and hence does not need additional gears
- The drawbacks are larger size, weight and cost because of the low speed design.

Inner-rotor

Outer-rotor

Tire

Tire

Rim

Rim

PM

Brake

Motor Winding

Wheel

Brake

Motor Winding

Wheel

Bearing

Encoder

PM

Bearing

Planetary gear set

Bearing

PM

Encoder

Wheel

Bearing

Motor Winding

Motor Winding

Brake

Brake

Wheel

PM

Rim

Rim

Tire

Tire

Figure 6a: Inner rotor *In Wheel* drive [1]

Figure 6b: Outer rotor *In Wheel* drive [1]

.

Considerations of EMs used in EVs

The requirements of EMs used in EVs are:

- Frequent start/stop
- High rate of acceleration and deceleration
- High torque low speed hill climbing
- Low torque cruising
- Very wide speed range of operation

The EMs for EVs are unique and their major differences with respect to industrial motors in load requirement, performance specification and operating environment are as follows:

- EV motors need to produce the maximum torque that is four to five times of the rated torque for acceleration and hill climbing, while industrial motors generally offer the maximum torque that is twice of the rated torque for overload operation
- EV motors need to achieve four to five times the base speed for highway cruising, while industrial motors generally achieve up to twice the base speed for constant power operation
- EV motors require high power density as well as good efficiency map (high efficiency over wide speed and torque ranges), while industrial motors are generally optimized to give high efficiency at a rated point.
- EV motors need to be installed in mobile vehicles with harsh operating conditions such as high temperature, bad weather and frequent vibration, while industrial motors are generally located in fixed places.

References:

[1]C. C. Chan and K. T. Chau, *Modern Electric Vehicle Technology*, Oxford Science Publication, 2001

Suggested Reading:

[1] I. Husain, *Electric and Hybrid Electric Vehicles*, CRC Press, 2003

[2] M. Ehsani, *Modern Electric, Hybrid Electric and Fuel Cell Vehicles: Fundamentals, Theory and Design*, CRC Press, 2005

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Module 6: A.C. Electrical Machines for Hybrid and Electric Vehicles

Lecture 17: Induction motors, their configurations and optimization for HEV/EVs

Fundamentals of Electrical Machines

Introduction

The topics covered in this chapter are as follows:

π. Electrical Machines in EVs and HEVs

θ. Physical Concepts of Torque production

ρ. Why Should the Number of Poles on Stator Equal to the Number of Poles on Rotor

σ. How Continuous Torque is Produced by a Motor τ. Rotating Magnetic Field

υ. How to Create the Second Magnetic Field

ϖ. Electrical and Mechanical Angle

Electrical Machines in EVs and HEVs

Vehicle propulsion has specific requirements that distinguish stationary and onboard motors. Every kilogram onboard the vehicle represents an increase in structural load. This increase structural load results in lower efficiency due to increase in the friction that the vehicle has to overcome. Higher efficiency is equivalent to a reduction in energy demand and hence, reduced battery weight.

The fundamental requirement for traction motors used in EVs is to generate propulsion torque over a wide speed range. These motors have intrinsically neither nominal speed nor nominal power. The power rating mentioned in the catalog and on the name plate of the motor corresponds to the maximum power that the drive can deliver. Two most commonly used motors in EV propulsion are Permanent Magnet (PM) Motors and Induction Motors (IM). These two motors will be investigated in detail in the coming lectures. However, before going into the details of these machines some basic fundamentals of electrical machines, such as torque production, are discussed in this chapter.

Physical Concepts of Torque Production

In **Figure 1a** a stator with 2 poles and a cylindrical rotor with a coil are shown. When only the stator coils are energized, stator magnetic flux is set up as shown in **Figure 1a**. The magnetic field for case when only the rotor coil is energized is shown in **Figure 1b**. In case when both the stator and rotor coils are energized, the magnetic resultant magnetic field is shown in **Figure 1c**. Since in this case the magnetic flux lines behave like stretched band, the rotor conductor experiences a torque in the direction shown in **Figure 1c**. From **Figure 1c** it can be seen that the stator ***S*** pole attracts the rotor ***N*** pole and repels the rotor ***S*** pole, resulting in clockwise torque. Similarly stator ***N*** pole attracts rotor ***S*** pole and repels rotor ***N*** pole, resulting again in clockwise torque.

Rotor

Stator

Figure 1a: Magnetic field when only stator is energised

Rotor

Stator

Figure 1b: Magnetic field when only rotor is energised

Figure 1c: Magnetic field when both stator and rotor are energised

The total torque is shown in **Figure 1c**. This torque is developed due to the interaction of stator and rotor magnetic fields and hence is known as ***interaction torque*** or ***electromagnetic torque***. The magnitude of the electromagnetic torque (T_{em}) or interaction torque is given by

$$T_{em} \propto (H_s)(H_r)\sin\delta \qquad (1)$$

where

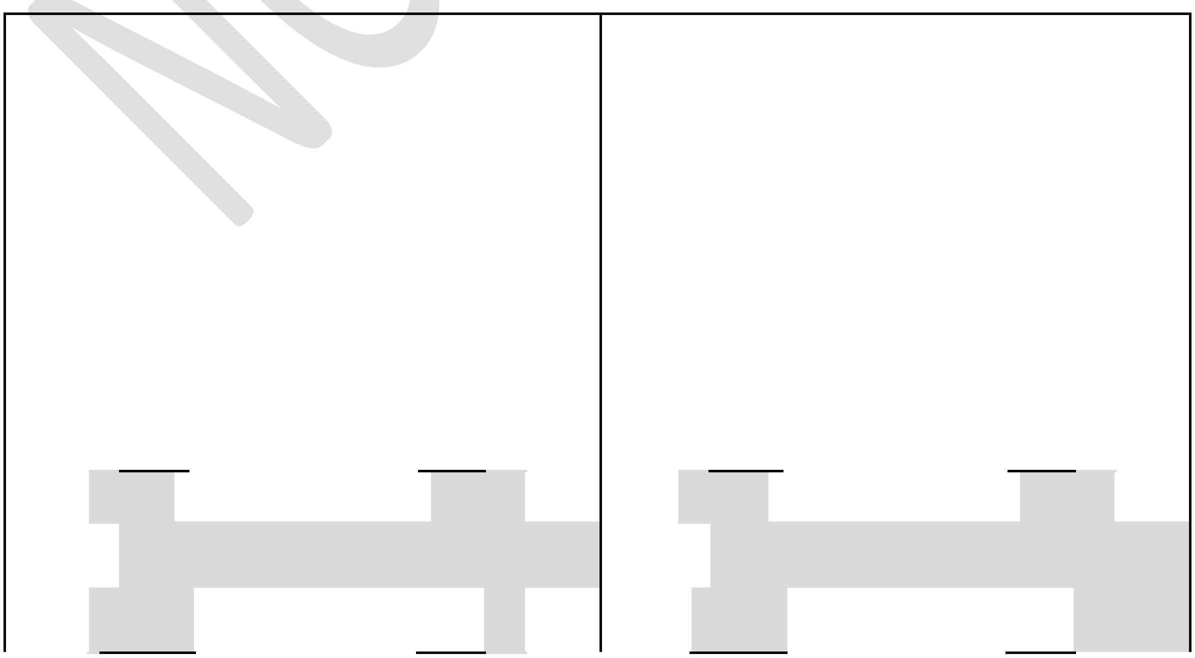
H_s is the magnetic field created by current in the stator winding

H_r is the magnetic field created by current in the rotor winding

• is the angle between stator and rotor magnetic field

Another configuration of the motor, with the flux lines, is shown in **Figure 2a**. Since the magnetic flux has a tendency to follow a minimum reluctance path or has a tendency to shorten its flux path, the rotor experiences an anti-clockwise torque. From **Figure 2a** it can be seen that the flux lines will have a tendency to align the rotor so that the reluctance encountered by them is reduced. The least reluctance position of the rotor is shown in

Figure 2b.



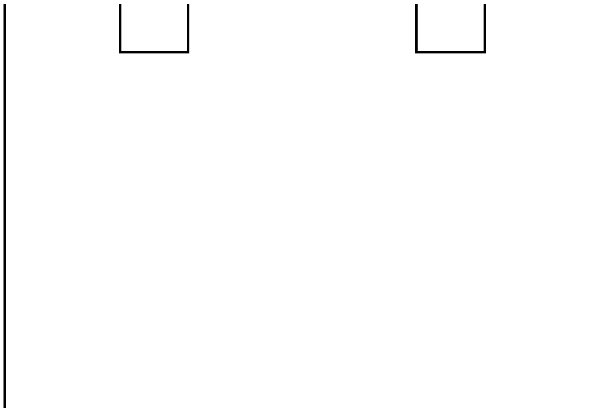
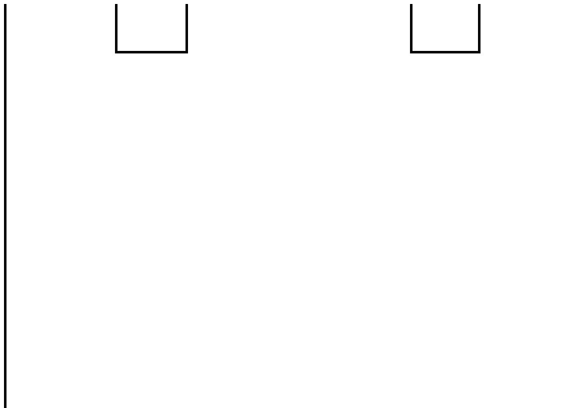
	

Figure 2a: Magnetic field when rotor axis is aligned with stator poles	**Figure 2b: Magnetic field when rotor axis is not aligned with stator poles**

To realign the rotor from the position shown in **Figure 2a** to position shown in **Figure 2b**, a torque is exerted by the flux lines on the rotor. This torque is known as the ***reluctance*** or ***alignment torque***.

Why Should the Number of Poles on Stator Equal to the Number of Poles on Rotor?

In the previous section it has been shown that to produce electromagnetic torque, the magnetic field produced by the stator has to interact with the magnetic field produced by the rotor. However, ***if the number of poles producing the stator magnetic field is not equal to the number of rotor poles producing the rotor magnetic field, then the net torque produced by the motor will be zero***. This is illustrated by the motor configuration shown in **Figure 3**. In this motor the stator has two poles (N_s , S_s) and the rotor has four poles (N_{r1} , S_{r1} , N_{r2} , S_{r2}). The angle between the stator poles is 180^o and the angle between the rotor poles is 90^o. From the arrangement shown in **Figure 3** it can be seen that the angle between N_{r1} and N_s is equal to the angle between N_{r2} and S . Hence, a repulsive force exists between N_{r1} and N_s in clockwise direction and an attractive force exists between N_{r2} and S in the anticlockwise direction. Both, the attractive and repulsive forces are of same magnitude and the resultant of these forces is zero.

N_{r1} S_{r1}

N_s S_s

Rotor

S_{r2} N_{r2}

Stator

Figure 3: Configuration of a motor with unequal number of stator and rotor poles

Now consider the pole pairs (N_s , $S_{r\ 2}$) and (S_s , S_{r1}), the angle between the pole pairs is same. Hence, the force of attraction between N_s and $S_{r\ 2}$ is same as the force of repulsion between S_s and S_{r1} and thus, the resultant force acting on the rotor is zero. Therefore, in this case no electromagnetic torque is developed.

From the above discussion it can be seen that the resultant electromagnetic torque developed due to two stator poles and 4 rotor poles is zero. This leads to the conclusion that ***in all rotating electric machines, the number of rotor poles should be equal to number of stator poles for electromagnetic torque to be produced***.

How Continuous Torque is Produced by a Motor

In the previous section it has been seen that to produce electromagnetic torque, following two conditions have to be satisfied:

- Both stator and rotor must produce magnetic field
- The number of magnetic poles producing the stator magnetic field must be same as the number of magnetic poles producing the rotor magnetic field.

Now an important question that arises is ***how to create continuous magnetic torque***? To produce continuous torque the magnetic field of the stator should rotate continuously. As a result, the rotor's magnetic field will chase the stator's magnetic field and this result in production of continuous torque. This phenomenon is illustrated in **Figures 4a-4d**. In **Figure 4** a two pole machine is depicted and the rotors magnetic field is created by the permanent magnets. It is assumed that the stator's magnetic field rotates at a speed of 60 revolutions per minute (60 rpm) which is equivalent 1 revolution per second (1rps). To start the analysis it is assumed that at time $t = 0$, the stator's magnetic field axis aligns itself with the $x -$ axis and the rotor's magnetic field axis makes an angle δ with the stator's magnetic axis (**Figure 4a**). At time $t = 0.25s$, the stator's magnetic field moves by 90^{o} and the rotor's magnetic field chases the stator's field and aligns as shown in **Figure 4b**. Similarly the locations of the magnetic field axis at time $t = 0.5s$ and $t = 0.75s$ are shown in **Figures 4c** and **Figure 4d**.

From the above discussion and observing **Figure 4** the following conclusions can be drawn:

- The rotor's magnetic field chases the stator's magnetic field.
- The angle (δ) between the stator's magnetic axis and the rotor's magnetic axis remains constant. Hence, the rotor's speed of rotation is same as that of the stator's magnetic field.

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However, an important question that still remains unanswered is ***How to create a rotating magnetic field?***

$y-axis$

S_s

S_r

δ

N_r

N_s

S_s

δ $x-a$ $x-ax$

N_r S_r

N_s

Figure 4a: Stator's magnetic field at time $t=0$

Figure 4b: Stator's magnetic field at time $t=0.25$

$y-axis$ $y-axis$

N_s

N_r

S_r

δ

S_s N_s

$x - axis$ $x - axis$

S_r N_r δ

S_s

Figure 4c: Stator's magnetic field at time $t = 0.5$

Figure 4d: Stator's magnetic field at time $t = 0.75$

.

Rotating Magnetic Field

To understand the rotations of magnetic field consider a 2-pole 3-phase stator as shown in **Figure 5a**. The three phase windings ***a, b*** and ***c*** are represented by three coils aa' , bb' and cc' . A current in phase ***a*** winding establishes magnetic flux directed along the magnetic axis of coil aa' . Similarly, the currents in phase ***b*** and ***c*** windings will create fluxes directed along the magnetic axes of coils bb' and cc' respectively. The three phase currents flowing the winding is shown in **Figure 5a**. At time instant **1**, the currents of each phase are

$$i_a = I_m \ ; \ i_b = -\frac{I_m}{2} \ ; \ i_c = -\frac{I_m}{2}$$

where (2)

I_m = maximum value of the current

Since, i_b and i_c are negative, crosses must be shown in coil-sides b' and c' and dots in the coil sides b and c .The right hand thumb rule gives the flux distribution as shown in **Figure 5b**. In **Figure 5b** and the following figures, the thicker line indicates higher magnitude to flux. The

At instant **2**, the currents are

$$i_a = \frac{I_m}{2} \ ; \ i_b = \frac{I_m}{2} \ ; \ i_c = -I_m \qquad (3)$$

The magnetic flux distribution created by the currents at instant **2** is shown in **Figure 5c**. Eventually at instant **3**, the currents are

$$i_a = -\frac{I_m}{2}; \; i_b = I_m; \; i_c = -\frac{I_m}{2} \quad (4)$$

The magnetic flux distribution created by the phase currents given by **equation 4** is shown in **Figure 5d**. From **Figure 5b** to **5c** it can be seen that the 2 poles produced by the resultant flux are seen to have turned 60^{o}. At other instants of time, i.e. as time elapses, the two poles rotate further. In this manner a rotating magnetic field is produced. The space angle traversed by a rotating flux is equal to the time angle traversed by currents.

After having discussed the production of rotating magnetic field, an important issue that still remains unresolved is: ***How to create the second magnetic field that will follow the rotating magnetic field created by the stator?*** This question is answered in following section.

Phase B
Phase A
Phase C
I_m
60^o 60^o
Instant 1 Instant 2 Instant 3

a'
c
b
b'
c'
a

Figure 5a: Three phase currents given to stator windings

Figure 5b: Magnetic flux at *instant 1*

a'
c
b

a'
c
b

b'

c'

a

Figure 5c: Magnetic flux at *instant 2*

b'

c'

a

Figure 5d: Magnetic flux at *instant 3*

How to Create the Second Magnetic Field

From **equation 1** it can be seen that to produce torque two magnetic fields are required. The rotating magnetic field created by the stator has been discussed in the previous section and this section deals with the generation of rotor magnetic field. There are multiple ways to produce the rotor magnetic field namely:

- α Having windings on the rotor and exciting then with dc current to produce magnetic field (known as ***Synchronous Machines***).
- α Having permanent magnets on the rotor to produce the rotor magnetic field (known as ***Permanent Magnet Synchronous Machines***).
- α Utilize the Faradays law of induction to induce electromotive force (e.m.f) in the rotor coils. The induced e.m.f will result in flow of current through the rotor conductors and these currents will produce a magnetic field. These machines are known as ***Induction Machines*** or ***asynchronous machines***.

Synchronous Machines

The general configuration of synchronous machine is shown in **Figure 6**. It can be seen from **Figure 6** that the rotor has a coil (denoted by a dot and a cross) and through this coil a dc current flows. Due to this dc current a pair of magnetic poles is created. The stator windings also create two magnetic fields that rotate with time and hence, the rotor's magnetic poles chase the stator's magnetic field and in the process electromagnetic torque is produced. The speed of rotation of rotor depends on the speed with which the stator's field rotates and hence, these machines are known as ***synchronous machine***.

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Permanent Magnet Synchronous Machines (PMSM)

In case of PMSM, the rotor field is created by permanent magnets rather than dc current passing through a coil (**Figure 7**). The principle of operation of PMSM is same as that of synchronous machine.

a' c b b' c' a

Figure 6: Synchronous machine

a' N c b b' c' S a

Figure 7: Permanent Magnet Synchronous machine

Induction Machine (IM)

Like synchronous machine, the stator winding of an induction machine is excited with alternating currents. In contrast to a synchronous machine in which a field winding on the rotor is excited with dc current, alternating currents flow in the rotor windings of an induction machine. In IM, alternating currents are applied to the stator windings and the rotor currents are produced by induction. The details of the working of the IMs are given in the following lectures.

After having discussed the general features of the electrical machines, the question that arises is***: how to analyse the machines?*** The analysis of electrical machines becomes simple by use of electrical equivalent circuits. The electrical equivalent circuits for the machines are discussed in the next section. One last concept that is relevant to electrical machines is principle of ***electrical*** and ***mechanical angle*** which is explained in the next section.

Electrical and Mechanical Angle

In **Figure 8**, it is assumed that the field winding is excited by a dc source and a coil rotates in the air gap at a uniform angular speed. When the conductor is aligned along $y - y'$ axis, the e.m.f induced is zero. Along $x - x'$ axis the induced e.m.f is maximum. In one revolution of the coil, the e.m.f induced is shown in **Figure 9**. If the same coil rotates in a 4 pole machine (**Figure 10**), excited by a dc source, the variation in the magnetic flux density and the induced e.m.f is shown in **Figure 11**. From **Figure 11** it can be seen that in one revolution of 360 mechanical degrees, 2 cycles of e.m.f (720 electrical degrees) are induced. The 720 electrical degrees in a 4 pole machine can be related to 360 mechanical degrees as follows

$$720 \text{ electrical degrees} = \frac{4}{2} \times (360 \text{ mechanical degrees}) \quad (5)$$

$$\Rightarrow \theta_{elec} = \frac{4}{2}\theta_{mech}$$

y'

x' x

0 60 120 180 240 300 360

Rotor Angle [^{0}]

y

Figure 8: A two pole machine

Figure 9: Induced e.m.f in the rotor coils of a two pole machine

.

0 60 120 180 240 300 360

Rotor Angle [^{o}]

Figure 10: A four pole machine

Figure11: Induced e.m.f in the rotor coils of a four pole machine

For a **P**-pole machine, **P/2** cycles of e.m.f will be generated in one revolution. Thus, for a **P** pole machine

$$\theta_{elec} = \frac{P}{2}\theta_{mech}$$

$$\Rightarrow \frac{d\theta_{elec}}{dt} = \frac{P}{2}\frac{d\theta_{mech}}{dt} \qquad (6)$$

$\zeta) \; \omega_{elec} = \frac{P}{2} \omega_{mech}$

In a 4 pole, in one revolution 2 cycles of e.m.f are generated. Hence, for a **P** pole machine, in one revolution **P/2** cycles are generated. For a **P**-pole machine, in one revolution per second, **P/2** cycles per second of e.m.f will be generated. Hence, for a **P** pole machine, in **n** revolutions per second $\frac{P}{2} \times n$ cycles/second are generated. The quantity cycles/second is the frequency ***f*** of the generated e.m.f. Hence,

$$f) = \frac{P}{2} \times n \text{ Hertz} \Rightarrow f = \frac{PN}{120} \text{ Hertz}$$

where (7)

N = the speed in rpm

Suggested Reading:

1. M. G. Say, *The Performance and Design of Alternating Current Machines*, CBS Publishers, New Delhi

2. S. J. Chapman, *Electric Machinery Fundamentals*, McGraw Hill, 2005

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Lecture 18: Induction motor drives, their control and applications in EV/HEVs

Induction Motor for EV and HEV Application

Introduction

The topics covered in this chapter are as follows:

ϑ Traction Motors

Κ Principle of Operation of Induction Motor (Mathematical Treatment)

Λ Principle of Operation of Induction Motor (Graphical Treatment)

Μ Fluxes and MMF in Induction Motor

Ν Rotor Action

Ο Rotor e.m.f and Equivalent Circuit

Π Complete Equivalent Circuit

Θ Simplification of Equivalent Circuit

Ρ Analysis of Equivalent Circuit

Σ Thevenin's Equivalent Circuit

Principles of Operation of Induction Motor (Mathematical Treatment)

In **Figure 1** a cross section of the stator of a three phase, two pole induction motor is shown. The stator consists of three blocks of iron spaced at 120° apart. The three coils are connected in Y and energized from a three phase system. When the stator windings are energized from a three phase system, the currents in the coils reach their maximum values at different instants. Since the three currents are displaced from each other by 120° electrical, their respective flux contributions will also be displaced by 120° electrical. Let a balanced three phase current be applied to the stator with the phase sequence ***A-B-C***

$$I_A = I_m \cos \omega t$$

$$I_B = I_m \cos\left(\omega t - \frac{2\pi}{3}\right) \quad (1)$$

$$I_C = I_m \cos\left(\omega t - \frac{4\pi}{3}\right)$$

The instantaneous flux produced by the stator will hence be

$$\phi_A = \phi_m \cos \omega t$$

$$\phi_B = \phi_m \cos\left(\omega t - \frac{2\pi}{3}\right) \quad (2)$$

$$\phi_C = \phi_m \cos\left(\omega t - \frac{4\pi}{3}\right)$$

The resultant flux at an angle θ from the axis of phase ***A*** is

$$\phi_T = \phi_A \cos(\theta) + \phi_B \cos(\theta - \frac{2\pi}{3}) + \phi_C \cos(\theta - \frac{4\pi}{3}) \qquad (3)$$

Substituting **equation 2** into **equation 3** gives

$$\phi_T = \phi_A \cos(\theta)\cos(\omega t) + \phi_B \cos(\theta - \frac{2\pi}{3})\cos(\omega t - \frac{2\pi}{3}) + \phi_C \cos(\theta - \frac{4\pi}{3})\cos(\omega t - \frac{4\pi}{3}) \qquad (4)$$

$$\Rightarrow \phi_T = \frac{3}{2}\phi_m \cos(\theta - \omega t)$$

From **equation** 4 it can be seen that the resultant flux has amplitude of 1.5 ϕ_m , is a sinusoidal function of angle θ and rotates in synchronism with the supply frequency.

Hence, it is called a ***rotating field***.

Figure 1: Cross section of a simple induction motor

Principles of Operation of Induction Motor (Graphical Treatment)

Let the synchronous frequency **ω** be 1rad/sec. Hence, the spatial distribution of resultant flux at t=0sec, t=60sec, t=120sec, t=180 sec, t=240sec and t=300sec and are shown in **Figure 2**. The explanation of the flux creation is as follows

χ At t=0, phase **A** is a maximum north pole, while phase **B** and phase **C** are weak south poles, **Figure (2a)**.

δ At t=60, phase **C** is a strong south pole, while phase **B** and phase **A** are weak north poles **Figure (2b)**.

ε At t=120, phase **B** is a strong north pole, while phase **A** and phase **C** are weak south poles **Figure (2c)**.

φ At t=180, phase **A** is a strong south pole, while phase **B** and phase **C** are weak north poles **Figure (2a)**.

γ At t=240, phase **C** is a strong north pole, while phase **A** and phase **B** are weak south poles **Figure (2e)**.

η At t=300, phase **B** is s strong south pole, while phase **C** and phase **A** are weak north poles **Figure (2f)**.

ϕm Phase A N

Angle around the periphery of motor

0 60 120 180 240 300 360

Phase B Phase C

ϕm

t=0 S

Figure 2a: Magnetic poles position at t=0

N

Phase A Phase B

0 60 120 180 240 300 360

S Phase C

Figure 2b: Magnetic poles position at t=60

Phase B N

N

ϕm Phase B Phase C

Angle around the periphery of motor

0 60 120 180 240 300 360

Phase C

Phase A

S

t=120

Figure 2c: Magnetic poles position at t=120

Angle around the periphery of motor

0 60 120 180 240 300 360

S

Phase A

Figure 2d: Magnetic poles position at t=180

.

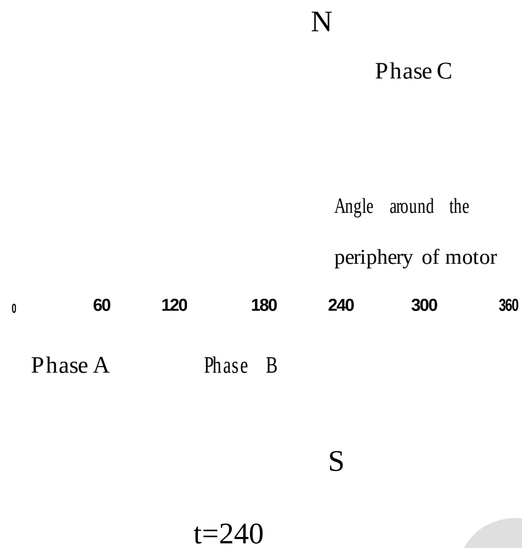


Figure 2e: Magnetic poles position at t=240

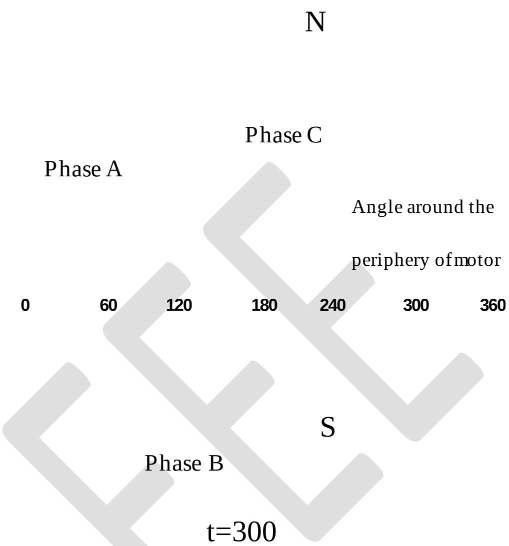


Figure 2f: Magnetic poles position at t=300

Fluxes and MMF in Induction Motor

Although the flux generated by each coil is only alternating flux, the combined flux contributions of the three coils, carrying current at appropriate sequential phase angles, produces a two pole rotating flux. The rotating flux produced by three phase currents in the stationary coils, may be linked to the rotating field produced by a magnet sweeping around the rotor (**Figure 3a**). The rotating field cuts the rotor bars in its anti clockwise sweep around the rotor. According to Lenz's law, the voltage, current and flux generated by the relative motion between a conductor and a magnetic field will be in a direction to oppose the relative motion. From **Figure 3a** it can be seen that the bars **a** and **b** are just under the pole centers and have maximum electromotive force (e.m.f) generated in them and this is indicated by large cross and dots. The bars away from the pole centers have reduced magnitude of generated e.m.fs and these are indicated by varying sizes of dots and crosses. If the rotor circuit is assumed purely resistive, then current in any bar is in phase with the e.m.f generated in that bar (**Figure 3a**). The existence of currents in the rotor circuit gives rise to rotor mmf F_2 , which lags behind airgap flux ϕ_m by a space angle of 90°. The rotor mmf causes the appearance two poles N_2 and S_2 .The relative speed between the poles N_1 , S_1 and the rotor poles N_2 , S_2 is zero. Rotating pole N_1 repels N_2 but attracts S_2 . Consequently the electromagnetic torque developed by the interaction of the airgap flux ϕ_m and the rotor mmf F_2 is in the same direction as that of the rotating magnetic field (**Figure 3b**). The space phase angle between F_2 and ϕ_m is called the load angle and for this case it is 90° (**Figure 3b**). The torque produce is given by

$$T_e = k\phi F_2 \sin(\frac{\pi}{2}) = k\phi F_2 \quad (5)$$

.

Rotating Airgap Flux

Rotating Airgap Flux

S2

N1 a b S1

N1 a b S1

N2

Figure 3a: Rotating airgap flux

Figure 3b: Rotor and Stator fields

In actual machine, the rotor bars are embedded in the iron, hence the rotor circuit has leakage reactance. Thus the rotor current in each bar lags behind the generated e.m.f in that bar by rotor power factor angle:

$$\theta_2 = \tan^{-1}\frac{x_2}{r_2} \qquad (6)$$

From **Figure 4** it is seen that bars **a** and **b** under the poles have a maximum generated e.m.fs. On account of the rotor reactance (x_2), the currents in these bars will be maximum only when the poles N_1 , S_1 have traveled through an angle θ_2 (**Figure 4**). The rotor current

generates rotor mmf F_2 is space displaced from the air gap flux ϕ_m by a load angle $\theta_2 + \frac{\pi}{2}$. The torque produced by the motor in this situation is

$$T_e = k\phi F_2 \sin\left(\frac{\pi}{2} + \theta_2\right) \quad (7)$$

.

Greater the value of x_2 , greater is the departure of load angle from its optimal value of $\frac{\pi}{2}$

and lesser is the torque. To generate a high starting torque, θ_2 should be made as small as possible and this is done by increasing rotor resistance r_2 .

S_2

Axis of stator field

N_1 a b S_1

Axis of rotor field

N_2

Figure 4: Axis of rotor and stator fields in case of rotor with inductance

Rotor Action

At standstill, rotor conductors are being cut by rotating flux wave at synchronous speed n_s . Hence, the frequency f_2 of the rotor e.m.f and current is equal to the input voltage frequency f_1 . When the rotor rotates at a speed of n_r rotations per second (r.p.s) in the direction of rotating flux wave, the relative speed between synchronously rotating flux and rotor conductors becomes ($n_s - n_r$) r.p.s, i.e.,

$$f_2 = \frac{P\,(n_s - n_r)}{2} \tag{8}$$

where P is the number of poles of the machine

Hence, the slip of the machine is defined as

$$s = \frac{n_s - n_r}{n_s} \tag{9}$$

Thus, the rotor frequency is defined as

$$f_2 = \frac{P \times s \times n_s}{2} = sf_1 \tag{10}$$

At standstill the rotor frequency is f_1 and the field produced by rotor currents revolves at a speed equal to $\frac{2f_1}{P}$ w.r.t. rotor structure. When the rotor rotates at a speed of nr, the rotor frequency is sf_1 and the rotor produced field revolves at a speed of $\frac{2(sf_1)}{P} = sn_s$ w.r.t. rotor structure. The rotor is already rotating at a speed of n_r w.r.t. stator. Hence, the speed of rotor field w.r.t. to stator is equal to the sum of mechanical rotor speed nr and rotor field speed sn_s w.r.t. rotor. Hence, the speed of the rotor field with respect to stator is given by

$$n_r + sn_s = n_s(1 - s) + sn_s = n_s \text{ r.p.s} \tag{11}$$

The stator and rotor fields are stationary with respect to each other at all possible rotor speeds. Hence, a steady torque is produced by their interaction. The rotor of an induction motor can never attain synchronous speed. If does so then the rotor conductors will be stationary w.r.t. the synchronously rotating rotor conductors and hence, rotor m.m.f. would be zero.

Rotor e.m.f and Equivalent Circuit

Let the rotor e.m.f. at standstill be E_2. When the rotor speed is $0.4n_s$, the slip is 0.6 and the relative speed between rotating field and rotor conductors is $0.6n_s$. Hence, the induced e.m.f., per phase, in the rotor is

$$0.6\, n_s \frac{E_2}{n_s} = 0.6E_2 \tag{12}$$

In general, for any value of slip s, the per phase induced e.m.f in the rotor conductors is equal to sE_2. The other quantities of the rotor are given as

The rotor leakage reactance at standstill is $x_2 = 2\pi f_1 L_2$ (13a)

The rotor leakage reactance at any slip s is $2\pi s f_1 L_2 = s x_2 \Omega$ (13b)

The rotor leakage impedance at standstill is $\sqrt{r_2^2 + x_2^2}$ (13c)

At any slip s rotor leakage impedance is $\sqrt{r_2^2 + (s x_2)^2}$ (13d)

The per phase rotor current at standstill is $\dfrac{E_2}{\sqrt{r_2^2 + x_2^2}}$ (13e)

The per phase rotor current at any slip s is $\dfrac{sE_2}{\sqrt{r_2^2 + (s x_2)^2}} = \dfrac{E_2}{\sqrt{\left(\dfrac{r_2}{s}\right)^2 + (x_2)^2}}$ (13f)

.

Based on **equation 13f** the equivalent circuit of the rotor is shown in **Figure 5**.

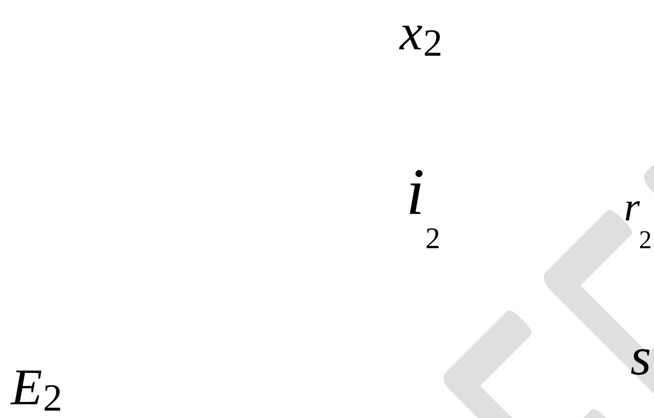


Figure 5: Equivalent circuit of rotor

Complete Equivalent Circuit

The rotating air gap flux generates back e.m.f. (E_1) in all the three phases of the stator. The stator applied terminal voltage V_1 has to overcome back e.m.f. E_1 and the stator leakage impedance drop:

$$V_1 = E_1 + I_1 (r_1 + jx_1) \qquad (14)$$

The stator current I_1 consists of following two components, I_1' and I_m . The component I_1' is the load component and counteracts the rotor m.m.f. The other component I_m creates the resultant air gap flux ϕ_m and provides the core loss. This current can be resolved into two components: I_c in phase with E_1 and I_ϕ lagging E_1 by 90^o. In the equivalent circuit of the stator shown in **Figure 6**, I_c and I_ϕ are taken into account by a parallel branch, consisting of core-loss resistance R_c in parallel to magnetizing reactance X_ϕ.

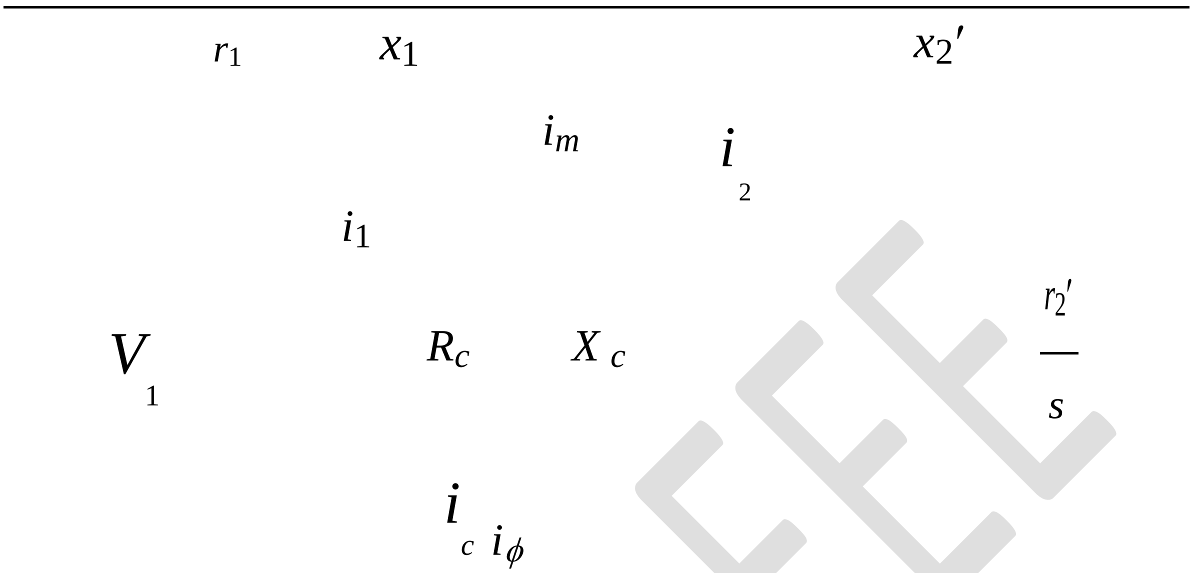


Figure 6: Equivalent circuit of stator

The rotor e.m.f. E_2 when referred to stator becomes

$$E_1 = \frac{N_1'}{N_2'} E_2 \quad (15)$$

where N_1' and N_2' are number of turns in the stator and rotor respectively

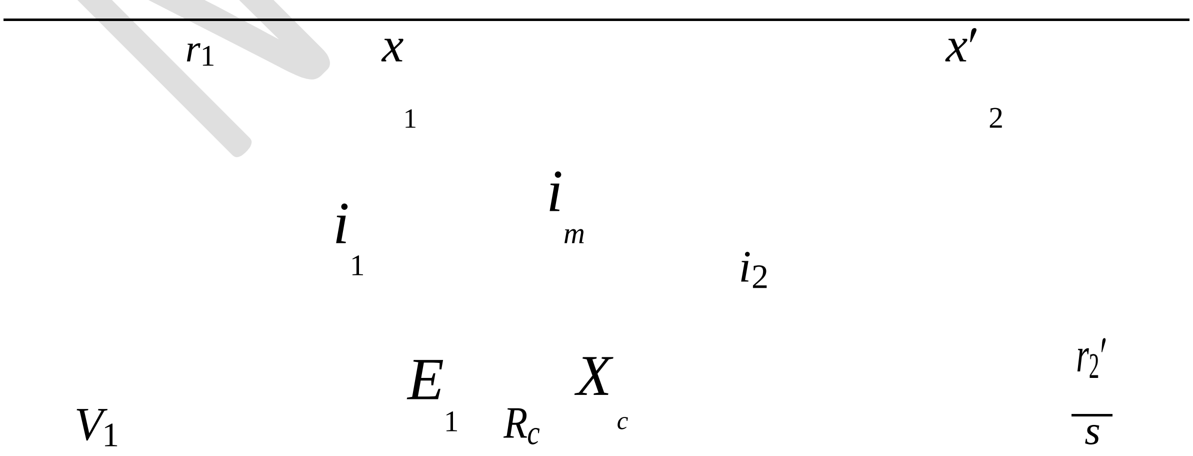
The rotor leakage impedance when referred to the stator is

$$Z_2' = \left(\frac{r_2}{s} + jx_2\right)\left(\frac{N_1'}{N_2'}\right)^2 = \frac{r_2'}{s} + jx_2' \quad (16)$$

where

$$r_2' = \left(\frac{N_1'}{N_2'}\right)^2 r_2 ; \quad x_2' = \left(\frac{N_1'}{N_2'}\right)^2 x_2$$

After referred the rotor quantities towards stator, the combined equivalent circuit of the machine is shown in **Figure 7**. For simplicity the prime notations will not be used in the further discussions and all the rotor quantities henceforth will be referred to the stator side. Moreover, all the quantities are at stator frequency.



$i_c \quad i_\phi$

Figure 7: Complete equivalent circuit of Induction Motor

Simplification Equivalent Circuit

The use of exact equivalent circuit is laborious; hence some simplifications are done in the equivalent circuit. Under normal operating conditions of constant voltage and frequency, core loss in induction motors is usually constant. Hence, the core loss component can be omitted from the equivalent circuit, **Figure 8**. However, to determine the shaft power, the constant core loss must be taken into account along with friction, windage and stray load losses. It should be noted that all the quantities used in the equivalent circuit are per phase quantities. Steady state performance parameters of the induction motor, such as current, speed, torque, losses etc. can be computed from the equivalent circuit shown in **Figure 8**.

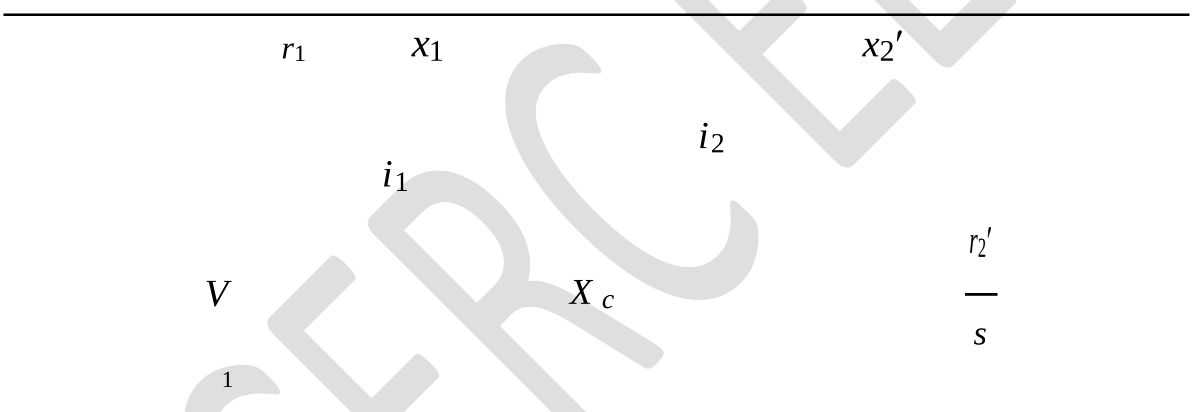


Figure 8: Simplified equivalent circuit of Induction Motor

Analysis of Equivalent Circuit

The total power transferred across the air gap (P_g) from the stator is

$$P_{gap} = n_{ph} i_2^2 \left(\frac{r_2}{s} \right)$$

$$= n_{ph}\left(i_2^2 r_2 + i_2^2 r_2 \left(\frac{1-s}{s} \right) \right) \quad (17)$$

where n_{ph} is the number of phases

Hence, the rotor Ohmic loses and the internal mechanical power are given as

$$P_{rotor} = n_{ph} i_2^2 r_2 = s P_{gap} \quad (18)$$

$$P_{mech} = n_{ph} i_2^2 r_2 \left(\frac{1-s}{s} \right) \quad (19)$$

The internal (gross or air gap or the electromagnetic torque) torque developed per phase is given by

$$T_e = \frac{P_{mech}}{\omega_r} = \frac{(1-s)P_{gap}}{(1-s)\omega_s} = \frac{P_{gap}}{\omega_s} \quad (20)$$

where ω_r is the rotor speed and ω_s is the synchronous speed

The output or the shaft power is

$$P_{shaft} = P_{mech} - Mechanical\ losses$$

$$or \tag{21}$$

$$P_{shaft} = P_{gap} - Rotor\ Ohmic\ losses - Mechanical\ losses$$

Thevenin's Equivalent Circuit of Induction Motor

When the torque-slip or power-slip characteristics are required, application of Thevenin's theorem to the induction motor equivalent circuit reduces the computation complexity. For applying Thevenin's theorem to the equivalent circuit shown in **Figure 8**, two points ***a***, ***b*** are considered as shown in **Figure 9**. From these points the voltage source V_1 is viewed and the equivalent voltage at point ***a*** and ***b*** is

$$V_{eq} = \frac{V_1 \left(jX_c \right)}{R_1 + j\left(X_1 + X_c \right)} \tag{22}$$

The equivalent impedance of the circuit as seen from points ***a*** and ***b*** is

$$Z_{eq} = \frac{\left(R_1 + jX_1 \right)\left(jX_c \right)}{R_1 + j\left(X_1 + X_c \right)} \tag{23}$$

For most induction motors ($X_1 + X_c$) is much greater than R_1 . Hence, R_1 can be neglected from the denominator of **equation 22** and **equation 23**. The simplified expression for V_{eq} and Z_{eq} are

$$V_{eq} = \frac{V_1 \left(jX_c \right)}{j\left(X_1 + X_c \right)} = \frac{V_1 X_c}{X_1 + X_c} \tag{24}$$

$$Z_{eq} = R_{eq} + jX_{eq} = \frac{R_1 X_c}{\quad} + \frac{jX_1 X_c}{\quad} \tag{25}$$

$eq \quad eq \quad eq$

$$X_1 + X_c \quad X_1 + X_c$$

From the Thevenin's equivalent circuit, the rotor current can be determined as

$$I_2 = \frac{V_{eq}}{\left(R_{eq} + \frac{r_2}{s}\right) + j\left(X_{eq} + X_2\right)} \tag{26}$$

The airgap torque produced by the motor is

$$T_e = \frac{n_{ph}}{\omega_s}\frac{V_{eq}^2}{\left(R_{eq}+\frac{r_2}{s}\right)^2+\left(X_{eq}+X_2\right)^2}\frac{r_2}{s}$$

$$= \frac{K_t}{\left(R_{eq}+\frac{r_2}{s}\right)^2+X^2}\frac{r_2}{s} \qquad (27)$$

where

$$K_t = \frac{n_{ph}V_{eq}^2}{\omega_s} \text{ and } X = X_2 + X_{eq}$$

A typical torque versus slip curve for IM obtained from **equation 27** is shown in **Figure 10**.

r_1 x_1 x_2

V_1 $\frac{r_2}{s}$

Figure 9: Thevenin's equivalent circuit

160
140
120
100
80
60
40
20
0

0 0.2 0.4 0.6 0.8 1

Slip

Figure 10: Torque vs. Slip curve of IM

Suggested Reading:

A M. G. Say, *The Performance and Design of Alternating Current Machines*, CBS Publishers, New Delhi

B S. J. Chapman, *Electric Machinery Fundamentals*, McGraw Hill, 2005

Lecture 19: Permanent magnet motors, their configurations and optimization

Control of Induction Motors

Introduction

The topics covered in this chapter are as follows:

- Π Speed Control of Induction Motor
- Θ Constant Volts/Hz control
- P Implementation of Constant Volts/Hz Control
- Σ Steady State Analysis of IM with Constant Volts/Hz Control

Speed Control of Induction Motor (IM)

Speed control of IM is achieved in the inverter driven IM by means of variable frequency. Besides the frequency, the applied voltage needs to be varied to keep the air gap flux constant. The induced e.m.f in the stator winding of an ac machine is given by

$$E_1 = 4.44 k_{w1} \phi_m f_s N_1 \qquad (1)$$

where

k_{w1} is the stator winding factor

ϕ_m is the peak airgap flux

f_s is the supply frequency

N_1 is the number of turns per phase in the stator

The stator applied terminal voltage V_1 (**Figure** 1) has to overcome back e.m.f. E_1 and the stator leakage impedance drop (refer Lecture 17):

$$V_1 = E_1 + i_1\,(r_1 + jx_1) \tag{2}$$

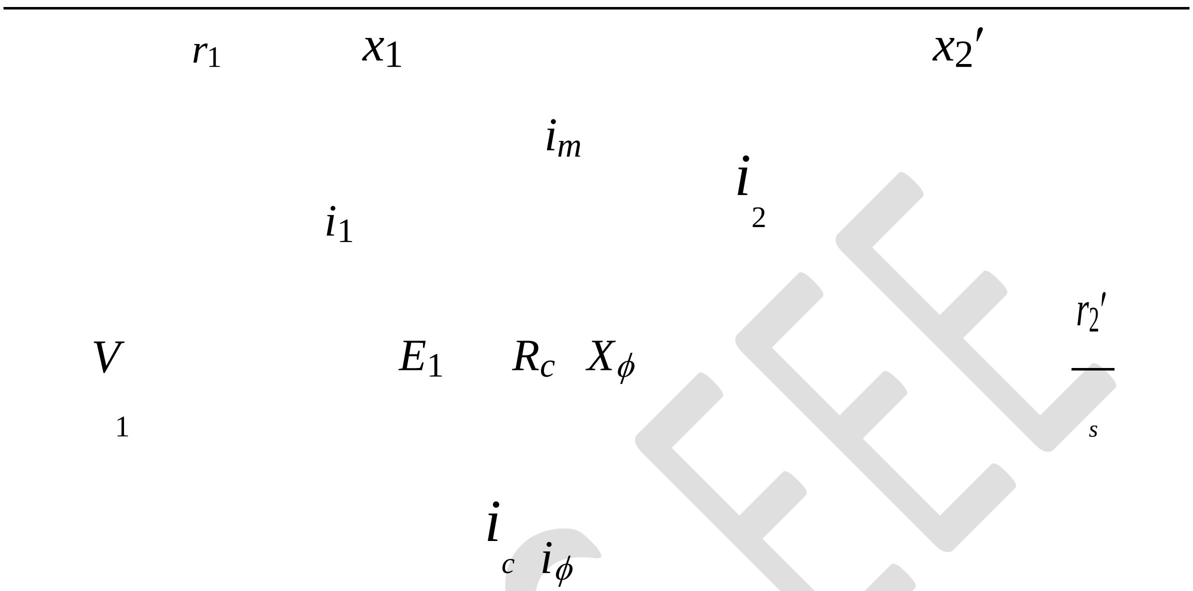


Figure 1: Equivalent circuit of IM

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If the stator impedance ($r_1 + jx_1$) is neglected, the induced e.m.f approximately equals the supply phase voltage. Hence,

$$V_1 \cong E_1 \qquad (3)$$

Substituting for E_1 from **equation 1** into **equation 2** gives the flux as

$$\phi_m \cong \frac{V_1}{K_b f_s} \qquad (4)$$

where

$k_b = 4.44 k_{w1} N_1$ is the flux constant

Since the factor k_b is constant, from **equation 4** it can be seen that ***proportional the flux is to the ratio between the supply voltage and frequency***. Hence,

$$\phi_m \propto \frac{V_1}{f_s} \propto k_{vf} \qquad (5)$$

where k_{vf} is the ration bewteen V_1 and f_s

From **equation 5**, it is seen that, to maintain the flux constant k_{vf} has to be maintained constant. Hence, whenever the stator frequency (f_s) is changed for speed control, the stator input voltage (V_1) has to be changed accordingly to maintain the airgap flux (ϕ_m) constant. A number of control strategies have been developed depending on how the voltage to frequency ratio is implemented:

- Constant volts/Hz control
- Constant slip-speed control
- Constant air gap control

- Vector Control

The constant volts/Hz strategy is explained in this lecture.

Constant Volts/Hz Control

The **equation 2** is converted into per unit (p.u) as

$$\frac{V_1}{V_b} = \frac{E_1 + I_1(r_1 + jx_1)}{V_b}$$

$= V_{1n} = E_{1n} + I_{1n}(r_{1n} + jx_{1n})$ where

$$V_{1n} = \frac{V_1}{V_b};\ I_{1n} = \frac{I_1}{I_b};\ r_{1n} = \frac{I_b r_1}{V_b}$$

$$E_{1n} = \frac{E}{V_b} = \frac{jX_\phi i_\phi}{V_b} = \frac{j\omega_s L_\phi i_\phi}{V_b} = \frac{j\omega_s L_\phi i_\phi}{\lambda_b \omega_b} = j\left(\frac{\lambda_\phi}{\lambda_b}\right)\left(\frac{\omega_s}{\omega_b}\right)$$

$$x_{1n} = \frac{I_b x_1}{V_b} = \frac{I_b \omega_s L_{1s}}{V_b} = L_{1n}\omega_{sn}$$

$V_b = \lambda_b \omega_b$

V_b is the base voltage

I_b is the base current

λ_b is the flux linkage (flux linkage is rate of change of flux with repect to time) Hence, **equation 2** in p.u form is written as

$$V_{1n} = E_{1n} + i_{1n}(r_{1n} + jx_{1n}) = i_{1n} r_{1n} + j\omega_{sn}\left(L_{1n} i_{1n} + \lambda_{\phi n}\right)$$

where

$$E_{1n} = jL_{\phi n}\omega_{sn} i_{\phi n} =$$

$$\lambda_{\phi n} = jL_{\phi n} i_{\phi n}$$

The magnitude of the input voltage (V_1) is given as

$$V_{1n} = (i_{1n} r_{1n})^2 + \omega_{sn}^2 (L_{1n} i_{1n}$$

$+ \lambda_{\phi n})^2$

(6)

(8)

(7)

.

For a constant air gap flux linkages of 1pu, the pu applied voltage vs. p.u stator frequency is shown in **Figure 2**. The values of r_{1n} and x_{1n} used to obtain the plot of **Figure 2** are 0.03 and 0.05 pu respectively.

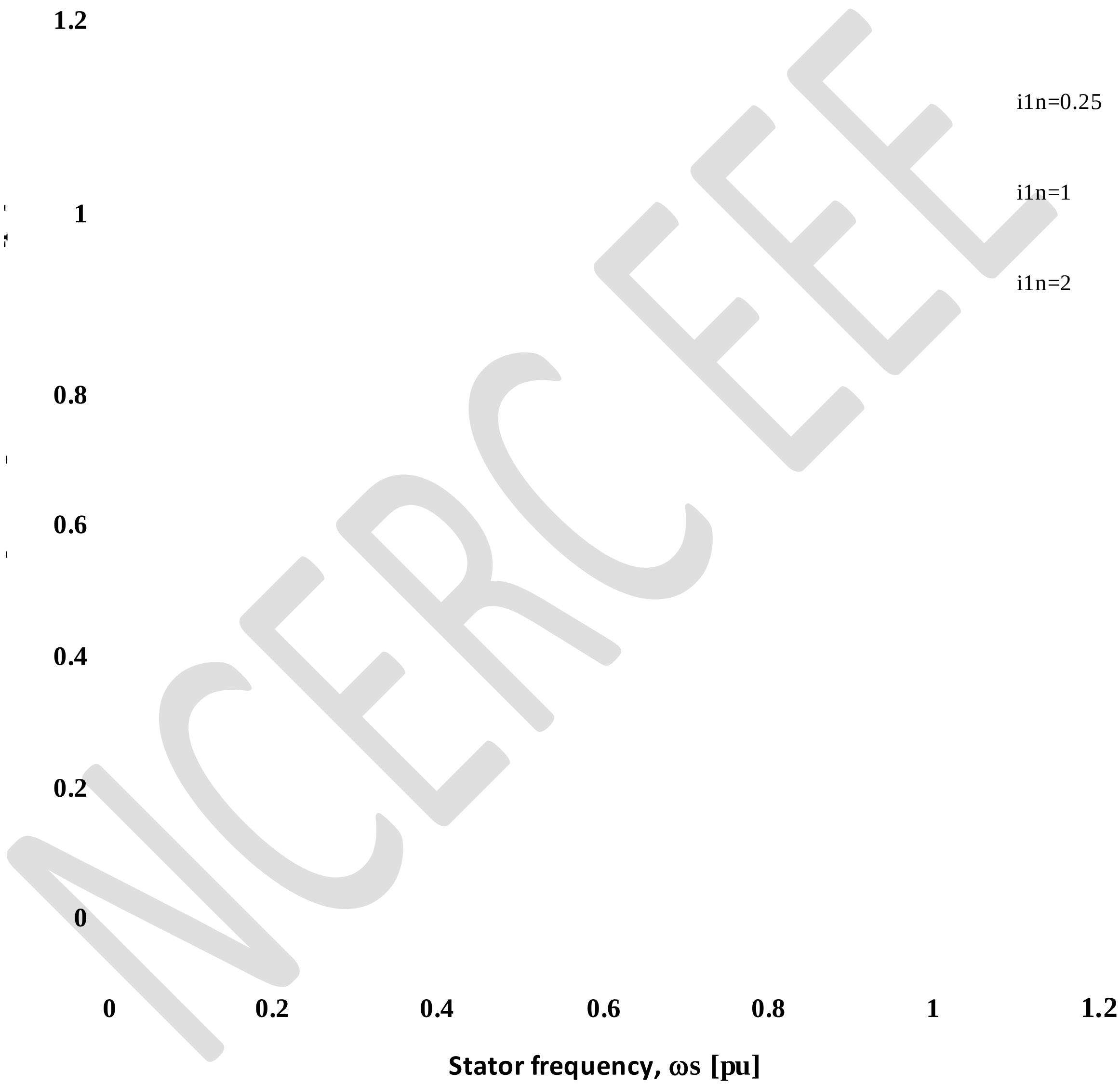

Figure 2: PU stator phase voltage vs. pu stator frequency

From **equation 8** it can be seen that the volts/Hz ratio needs to be adjusted in dependence on the frequency, the air gap flux magnitude, the stator impedance and the magnitude of the stator current. The relationship between the applied phase voltage and the frequency is written as

$$V_{1n} = V_{on} + k_{vf}\, f_{sn} \qquad (9)$$

From **equation 8** the parameters V_o and k_{vf} is obtained as

$$V_{on} = i_{1n}\, r_{1n}$$

$$k_{vf} = \omega_{sn} \left(\lambda_{\phi n} + L_{1n}\, i_{1n} \right) \qquad (10)$$

The parameter V_o is the offset voltage required to overcome the stator resistive drop. In case the IM is fed by a DC-AC converter, the fundamental r.m.s phase voltage for 180^o conduction is given by (refer **Lecture 15**):

$$V_1 = \frac{v_{as}}{\sqrt{2}} = \frac{2}{\pi}\frac{V_{dc}}{\sqrt{2}} = 0.45V_{dc} \qquad (11)$$

where

V_{dc} is the input dc voltage to DC-AC converter

The **equation 11** can be written in pu form as

$$V_{1n} = \frac{V_1}{V_b} = 0.45V_{dcn} \qquad (12)$$

Substituting the value of V_{1n} into **equation 9** gives

$$0.45V_{dcn} = V_{on} + k_{vf}\, f_{sn}$$

Implementation of Constant Volts/Hz Control

The implementation of volts/Hz strategy is shown in **Figure 3**.

AC
V_{in} DC
V_{dc}
IM
DC DC

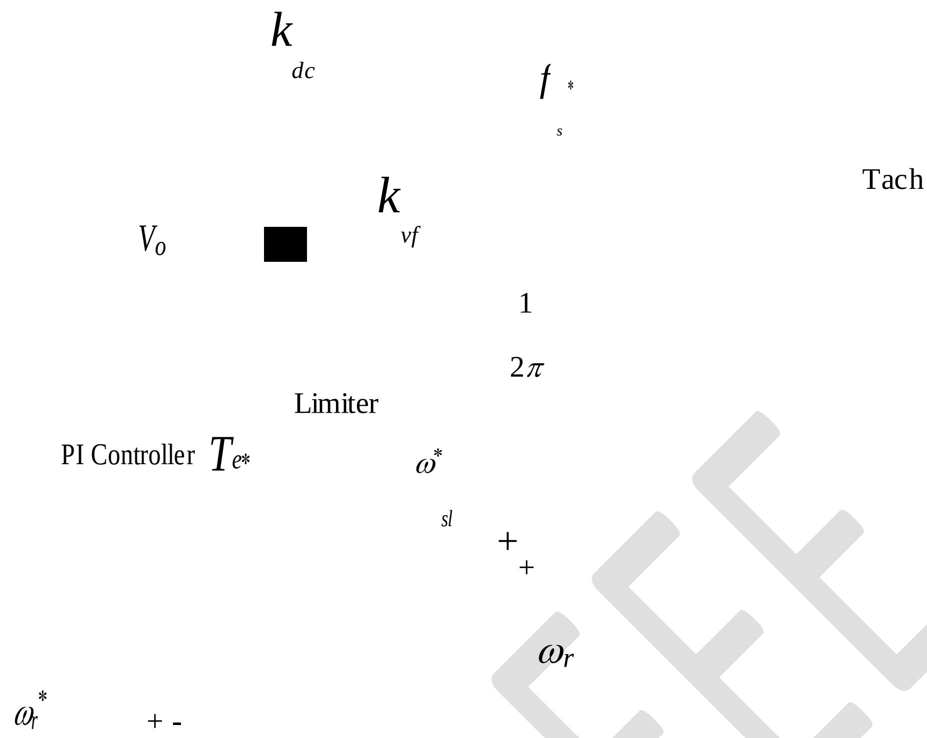

Figure 3: Closed loop induction motor drive with constant volts/Hz control strategy [1]

The working of the closed loop control shown in **Figure 3** is as follows:

- The actual rotor speed (ω_r) is compared with its desired value ω_r^* and the error is passed through a PI controller.
- The output of the PI controller is processed through a limiter to obtain the slip-speed command ω_{sl}^* . The limiter ensures that ω_{sl}^* is within the maximum allowable slip speed of the induction motor.

[3] The slip speed command is added to electrical rotor speed ω_r to obtain the stator frequency command f_s^* .

[4] The frequency command f_s^* is enforced in the inverter and the corresponding dc link voltage (V_{dc}) is controlled through the DC-DC converter.

[5] The offset voltage V_1^* is added to the voltage proportional to the frequency and multiplied by k_{dc} to obtain the dc link voltage.

Steady State Performance of IM with Constant Volts/Hz Control

The steady state performance of the constant-volts/Hz controlled induction motor is computed by using the applied voltage given in **equation 9**. Using the equivalent circuit of IM, the following steps are taken to compute the steady state performance:

[5] Start with a minimum stator frequency and a very small slip

[6] Compute the magnetization, core-loss, rotor and stator phase current

[7] Calculate the electromagnetic torque, power, copper and core losses

[8] Calculate the input power factor and efficiency.

[9] Increase the slip and go to ***step b*** unless maximum desired slip is reached.

[10] Increase the stator frequency and go to ***step a*** unless maximum desired frequency is reached.

In **Figure 4** the characteristics of volts/Hz control of an IM is shown. The parameters of the IM are as follows:

Applied stator line to line voltage $V_{ll} = 200V$

Frequency of applied voltage $f_s = 50Hz$

Rated Output Power $P_{out} = 3kW$

Stator resistance $r_1 = 0.3\Omega$

Stator leakage inductance $L_1 = 0.001H$

Rotor resistance $r_2 = 0.2\Omega$

Rotor leakage inductance $L_2 = 0.0015H$

Efficiency $\eta = 0.8$

Power factor $pf = 0.85$

.

Connection of phases: Y

Based on the above parameters of the motor, the base quantities are determined as follows:

Base speed $\omega_{base} = 2\pi f_S = 2\times \pi \times 50 = 314.16 \ rad/s$

Base voltage $V_{base} = V_{ph} = \dfrac{200}{\sqrt{3}} = 115.47 \ V$

Base power P_{base} = Rated Output Power = 3000 W

Base current $I_{base} = \dfrac{P_{base}}{3\times V_{base} \times \eta \times pf \times V_{base}} = 12.74 \ A$

Base Torque $T_{base} = \dfrac{P_{base}}{\omega_{base}} = \dfrac{3000}{314.16} = 9.55 \ Nm$

After having calculated the base values, the torque produced by the IM is calculated using the following expression (**equation 27** of Lecture 17):

$$T_e = \frac{n_{ph}}{\omega_s} \frac{V_{eq}^2}{\left(R_{eq} + \frac{r_2}{s}\right)^2 + \left(X_{eq} + X_2\right)^2} \frac{r_2}{s} \qquad (13)$$

The pu torque T_{en} is given by

$$T_{en} = \frac{T_e}{T_{base}} \qquad (14)$$

In order to obtain the curve shown in **Figure 4**, the torque is calculated for different values of slip and frequency as described algorithm above. Using the constant volts/Hz control, the IM can be operated up to rated frequency. However, if it is required to operate the motor beyond rated speed then ***Flux weakening operation*** is used.

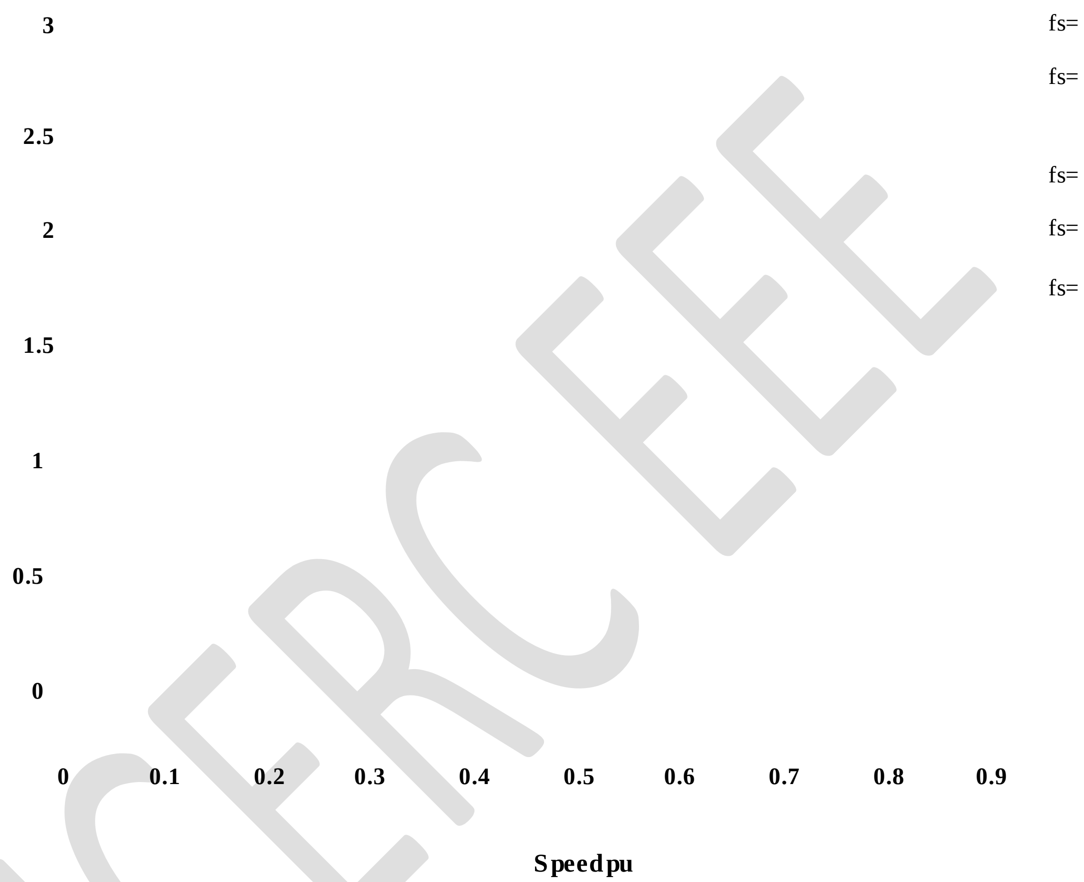

Figure 4: Torque vs. Speed Curve for Constant volts/Hz control of IM

From **Figure 4** the following points can be observed:

- As the frequency of the stator input voltage increases, the maximum speed of the motor increases.
- With increase in frequency, the maximum torque produced by the motor also increases.

$V_o = 7i_{1n} r_{1n}$ is shown. It

- The starting torque (torque at zero speed) does not vary much with increase in frequency.

In **Figure 5** the Torque vs. Speed curve for higher offset voltage can be seen that using a higher offset voltage:

- the starting torque has increased.
- the maximum torque produced by the motor at different frequencies is almost constant.

Here the motor is operated at 10Hz between the rotor speeds ω_{r1} and ω_{r2}, at 20 Hz between ω_{r2} and ω_{r3} and so on. With this operation constant torque is maintained almost up to rated speed.

4-

$f_s = 10Hz$ $f_s = 20Hz$ $f_s = 30Hz$ $f_s = 40Hz$ $f_s = 50Hz$

3-

2

1

ω_{r1} ω_{r2} ω_{r3} ω_{r4} ω_{r5}

0

Speed pu

Figure 5: Constant Torque vs. Speed Curve for Constant volts/Hz control of IM

Utilizing the second point, a constant torque can be obtained from starting condition up to rated speed as shown in **Figure 6**.

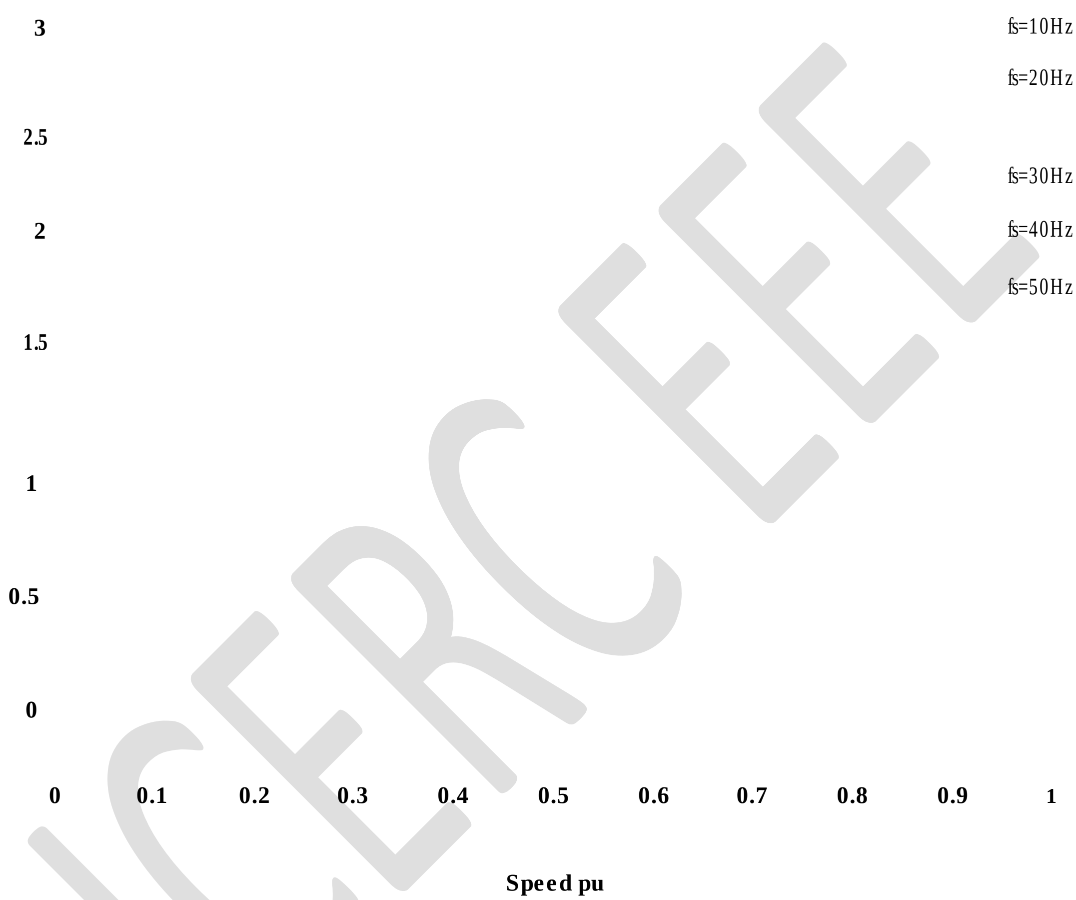

Figure 6: Torque vs. Speed Curve for Constant volts/Hz control of IM at higher offset voltage

The power factor vs. slip, stator current vs. slip curves and torque vs. slip for constant volts/Hz control are shown in **Figure 7**, **8** and **9** respectively. From **Figures 7-9** the following can be observed:

- As the slip increases (speed decreases) the power factor of the motor decreases. It attains a maximum value at a small slip (s_{pf}) value and then drops sharply.

- As the frequency increases the slop of the power factor between s_{pf} and unity slip increases.

- For any given slip, the magnitude of the stator current increases as the frequency increases. The magnitude of torque at a given slip also increases with increase in slip with the exception of unity slip (starting condition).

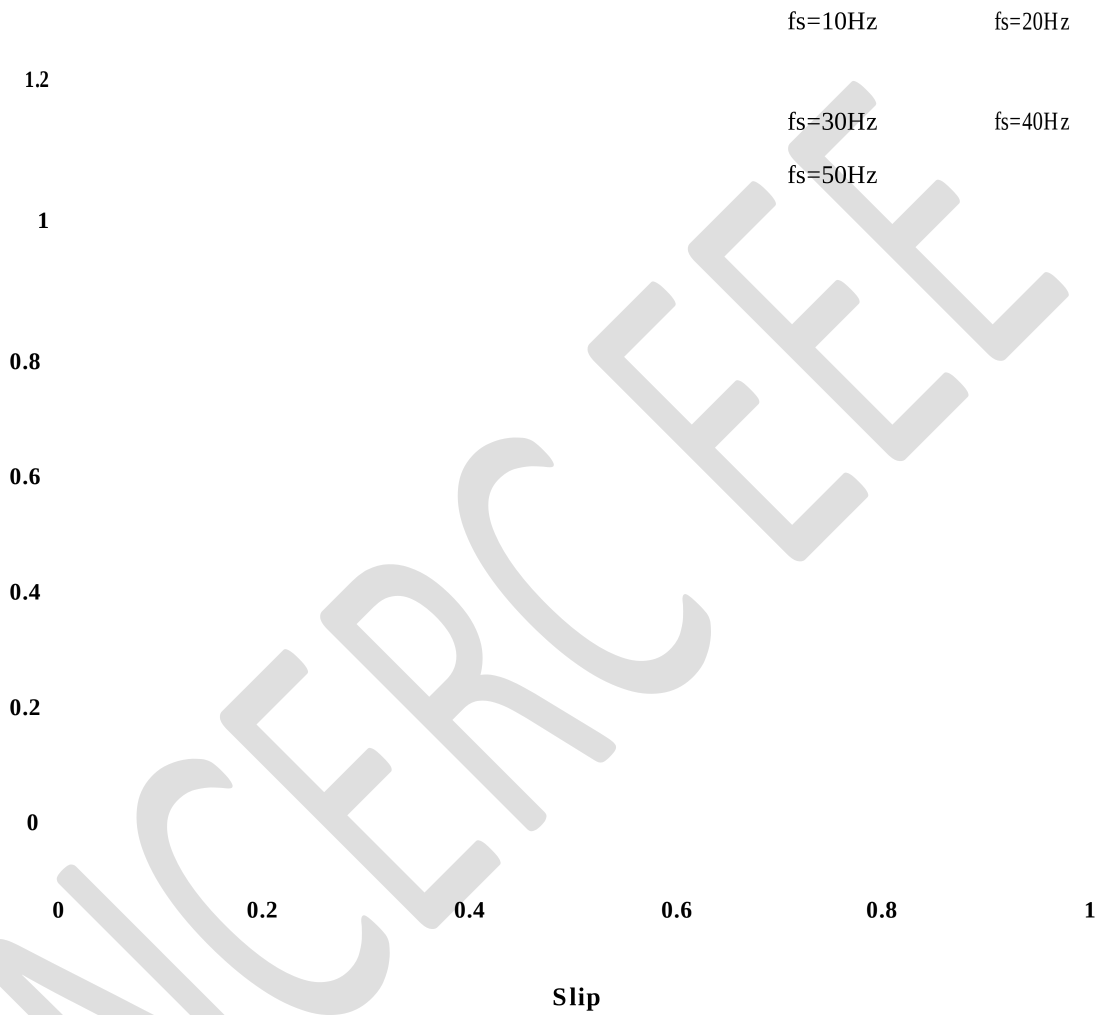

Figure 7: Power factor vs. Slip Curve for Constant volts/Hz control of IM

fs=10Hz fs=20Hz

fs=30Hz fs=40Hz

fs=50Hz

90

80

70

60

50

40

30

20

10

0

0 0.2 0.4 0.6 0.8 1

Slip

Figure 8: Stator Current vs. Slip Curve for Constant volts/Hz control of IM

1

0.5

3

fs=10Hz

fs=20Hz

0

2.5

fs=30Hz

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

2

fs=40Hz

Slip

fs=50hz

1.5

Figure 9: Torque vs. Slip Curve for Constant volts/Hz control of IM

References:

- R. Krishnan, *Electric motor drives: modeling, analysis, and control*, Prentice Hall, 2001

Lecture 20: Permanent magnet motor drives, their control and applications in EV/HEVs

Modeling of Induction Motor

Introduction

The topics covered in this chapter are as follows:

- Voltage Relations of Induction Motor
- Torque Equation in Machine Variables
- Equation of Transformation for Stator Variables
- Equation of Transformation for Rotor Variables
- Voltage and Torque Equations in Arbitrary Reference Frame Variables

Voltage Relations of Induction Motor

A 2 pole, 3 phase, Y connected symmetrical IM is shown in **Figure 1**. The stator windings are identical with N_s number of turns and the resistance of each phase winding is r_s . The rotor windings, may be wound or forged as squirrel cage winding, can be approximated as identical windings with equivalent turns N_r and resistance r_r . The air gap

of IM is uniform and the stator and rotor windings are assumed to be sinusoidally distributed. The sinusoidal distribution of the windings results in sinusoidal magnetic field in the air gap.

.

b_s axis

a_s'

b_r axis

c_s

a_r'

c_r

a_r axis

c_r'

a_s axis

b_r'

a_r

b_s'

c_s'

a_s

c_s axis

c_r axis

Figure 1: Two pole, 3 phase, wye-connected symmetrical induction motor

Since the windings are symmetric, the self inductances of stator windings are equal; that is

$$L_{asas} = L_{bsbs} = L_{cscs}$$

$$L_{asas} = L_{ls} + L_{ms}$$

where

L_{ls} is the leakage inductance of phase A or B or C of stator winding (1)

L_{ms} is the stator magnetizing inductance

L_{asas} , L_{bsbs} , L_{cscs} are the self inductances of stator phases

Similarly, the mutual inductances between the phases of the stator wind are same; that is

$$L_{asbs} = L_{bscs} = L_{csas} = -\frac{1}{2} L_{ms} \quad (2)$$

Based on the above discussion, the rotor self inductances and the mutual inductances between the rotor phases are

$$L_{arar} = L_{brbr} = L_{crcr}$$

$$L_{arar} = L_{lr} + L_{mr} \tag{3}$$

where

L_{lr} is the leakage inductance of phase a of rotor winding L_{mr} is the rotor magnetizing inductance

$$L_{arbr} = L_{brcr} = L_{crar} = -\frac{1}{2} L_{mr} \tag{4}$$

There exists mutual inductance between the stator and rotor windings. This mutual inductance is not constant because as the rotor rotates, the angle between the stator and rotor windings changes. Hence, the mutual inductance between the stator and rotor windings can be expressed as:

$$L_{asar} = L_{bsbr} = L_{cscr} = L_{sr} \cos(\theta_r)$$

$$L_{asbr} = L_{bscr} = L_{csar} = L_{sr} \cos\left(\theta_r + \frac{2\pi}{3}\right) \tag{5}$$

$$L_{ascr} = L_{bsar} = L_{csbr} = L_{sr} \cos\left(\theta_r - \frac{2\pi}{3}\right)$$

The voltage equations for the IM shown in **Figure 1** are

$$v_{as} = r_s i_{as} + \frac{d\lambda_{as}}{dt}$$

$$v_{bs} = r_s i_{bs} + \frac{d\lambda_{bs}}{dt}$$

$$v_{cs} = r_s i_{cs} + \frac{d\lambda_{cs}}{dt}$$

$$v_{ar} = r_r i_{ar} + \frac{d\lambda_{ar}}{dt} \quad (6)$$

$$v_{br} = r_r i_{ar} + \frac{d\lambda_{ar}}{dt}$$

$$v_{cr} = r_r i_{cr} + \frac{d\lambda_{cr}}{dt}$$

The **equation 4** can be expressed in matrix form as

$$v_{abs} = r_s i_{abcs} + p\lambda_{abcs}$$
$$v_{abr} = r_r i_{abcr} + p\lambda_{abcr}$$

where

$$(v_{abcs})^T = [v_{as} \quad v_{bs} \quad v_{cs}]$$
$$(v_{abcr})^T = [v_{ar} \quad v_{br} \quad v_{cr}]$$
$$(i_{abcs})^T = [i_{as} \quad i_{bs} \quad i_{cs}] \qquad (7)$$
$$(i_{abcr})^T = [i_{ar} \quad i_{br} \quad i_{cr}]$$
$$p = \frac{d}{dt}$$

In the above equation the subscript ***s*** refers to ***stator*** and ***r*** refers to ***rotor***. For a magnetically linear system, the flux linkages may be expressed as

$$\begin{bmatrix} \lambda_{abcs} \\ \lambda_{abcr} \end{bmatrix} = \begin{bmatrix} L_s & L_{sr} \\ (L_{sr})^T & L_r \end{bmatrix} \begin{bmatrix} i_{abcs} \\ i_{abcr} \end{bmatrix} \qquad (8)$$

The stator and rotor windings inductances consist of self and mutual inductances and is represented as

$$L_s = \begin{bmatrix} L_{ls} + L_{ms} & -\frac{1}{2}L_{ms} & -\frac{1}{2}L_{ms} \\ -\frac{1}{2}L_{ms} & L_{ls} + L_{ms} & -\frac{1}{2}L_{ms} \\ -\frac{1}{2}L_{ms} & -\frac{1}{2}L_{ms} & L_{ls} + L_{ms} \end{bmatrix} \qquad L_r = \begin{bmatrix} L_{lr} + L_{mr} & \cdots \\ -\frac{1}{2}L_{mr} & \cdots \\ -\frac{1}{2}L_{mr} & \cdots \end{bmatrix}$$

$$\begin{matrix} -\frac{1}{2}L_{ms} & L_{ls}+L_{ms} \end{matrix} \qquad (9)$$

$$\begin{bmatrix} & -\frac{1}{2}L_{mr} & -\frac{1}{2}L_{mr} \\ & L_{lr}+L_{mr} & -\frac{1}{2}L_{mr} \\ & -\frac{1}{2}L_{mr} & L_{lr}+L_{mr} \end{bmatrix} \qquad (10)$$

$$L_{sr} = L_{sr}\begin{bmatrix} \cos(\theta_r) & \cos(\theta_r+\frac{2\pi}{3}) & \cos(\theta_r-\frac{2\pi}{3}) \\ \cos(\theta_r-\frac{2\pi}{3}) & \cos(\theta_r) & \cos(\theta_r+\frac{2\pi}{3}) \\ \cos(\theta_r+\frac{2\pi}{3}) & \cos(\theta_r-\frac{2\pi}{3}) & \cos(\theta_r) \end{bmatrix} \qquad (11)$$

The expression of voltage equation becomes convenient if all the rotor variables are referred to stator windings using the turns ratio:

$$i'_{abcr} = \frac{N_r}{N_s} i_{abcr}$$

$$v'_{abcr} = \frac{N_r}{N_s} v_{abcr} \qquad (12)$$

$$\lambda'_{abcr} = \frac{N_r}{N_s} \lambda_{abcr}$$

The magnetizing and mutual inductances are associated with the same magnetic flux path; hence,

$$L_r' = \left(\frac{N_s}{N_r}\right)^2 L_r; \quad L'_{lr} = \left(\frac{N_s}{N_r}\right)^2 L_{lr}$$

$$\therefore L_r' = \begin{bmatrix} L'_{lr} + L_{ms} & -\frac{1}{2}L_{ms} & -\frac{1}{2}L_{ms} \\ -\frac{1}{2}L_{ms} & L'_{lr} + L_{ms} & -\frac{1}{2}L_{ms} \\ -\frac{1}{2}L_{ms} & -\frac{1}{2}L_{ms} & L'_{lr} + L_{ms} \end{bmatrix} \qquad (13)$$

Using **equation 13**, the flux linkages given by **equation 8** can be expressed as

$$\begin{bmatrix} \lambda_{abcs} \\ \end{bmatrix} = \begin{bmatrix} L_s & L'_{sr} \\ (\ )^T & \end{bmatrix}$$

$$\left\lfloor \lambda'_{abcr} \right\rfloor \left\lfloor (L'_{sr})^T \quad L'_r \right\rfloor \left\lfloor i'_{abcr} \right\rfloor$$

and using **equation 14**, the voltage equation (**equation 7) can be written as**

$$\begin{bmatrix} v_{abcs} \\ v'_{abcr} \end{bmatrix} = \begin{bmatrix} r_s + pL_s & pL'_{sr} \\ p(L'_{sr})^T & r'_r + pL'_r \end{bmatrix} \begin{bmatrix} i_{abcs} \\ i'_{abcr} \end{bmatrix} \qquad (14)$$

where $r'_r = \left(\frac{N_s}{N_r}\right)^2 r_r$ (15)

Torque Equation in Machine Variables

The conversion into machine variables can be done using the principle of magnetic energy. In a machine, the stored magnetic energy is the sum of the self-inductance of each winding. The energy stored due to stator winding is:

$$W_s = \frac{1}{2}(i_{abcs})^T (L_s - L_{ls} I) i_{abc}$$

where (16)

I is the identity matrix

Similarly, the energy stored due to rotor winding is

$$W_r = \frac{1}{2}(i'_{abcr})^T (L'_r - L'_{lr} I) i'_{abcr} \qquad (17)$$

The energy stored due to mutual inductance between the stator and rotor windings is

$$W_{sr} = (\mathrm{i}_{abcs})^T \mathrm{L}'_{sr}\, \mathrm{i}'_{abcr} \tag{18}$$

Hence, the total energy stored in the magnetic circuit of the motor is

$$W_f = W_s + W_{sr} + W_r$$

$$= \frac{1}{2}(\mathrm{i}_{abcs})^T(\mathrm{L}_s - \mathrm{L}_{ls}\mathrm{I})\,\mathrm{i}_{abcs} + (\mathrm{i}_{abcs})^T \mathrm{L}'_{sr}\,\mathrm{i}'_{abcr} + \frac{1}{2}(\mathrm{i}'_{abcr})^T(\mathrm{L}'_r - \mathrm{L}'_{lr}\mathrm{I})\,\mathrm{i}'_{abcr} \tag{19}$$

Since the magnetic circuit of the machine is assumed to be linear (the of **B** vs. **H**), the stored energy in the magnetic field W_f is equal to the co-energy W_{co} . The electromagnetic torque produced by the IM is given by

$$T_e = \left(\frac{P}{2}\right)\frac{\partial W_{co}}{\partial \theta_r} = \left(\frac{P}{2}\right)\frac{\partial W_f}{\partial \theta_r} \tag{20}$$

where θ_r is the rotor angle at any given point of time.

Since the inductances L_s, L_{ls}, L'_{lr}, L'_r are not functions of θ_r and only L'_{sr} is a function of θ_r (**equation 11**), substituting the **equation 19** into **equation 21** gives

$$T_e = \left(\frac{P}{2}\right)(\mathrm{i}_{abcs})^T \frac{\partial}{\partial \theta_r}\left[\mathrm{L}'_{sr}\right]\mathrm{i}'_{abcr} \tag{21}$$

Linear Transformations

From **equation 6** it can be seen that in order to study the dynamic behaviour of IM, a set of ***six*** equations have to be solved. If the number of equations to be solved is reduced, the computational burden will be reduced. In order to reduce the number of equations ***linear transformation*** is carried out. It is very common to use linear transformation to solve problems and one of the most common examples of it is ***logarithm***. The logarithms are used to multiply or divide two numbers. Similarly, the Laplace transform is also a linear transformation. It transforms the time-domain equations to s – domain equation and after manipulations, one again gets the required time-domain solution. The process of referring secondary quantities to primary or primary to secondary in a transformer is also equivalent to a linear transformation. It should be noted that ***the transformation from old to new set of variables is carried out for simplifying the calculations***.

Linear transformations in electrical machines are usually carried out to obtain new equations which are fewer in number or are more easily solved. For example, a three phase machine are more complicated because of the magnetic coupling amongst the three phase windings as seen from **equation 11**, but this is not the case after the transformation.

Transformation from Three Phases to Two Phases (a, b, c to α , β, 0)

A symmetrical 2 pole, 3 phase winding on the rotor is represented by three coils ***A, B, C*** each of N_r turns and displaced by 120^o is shown in **Figure 2**.

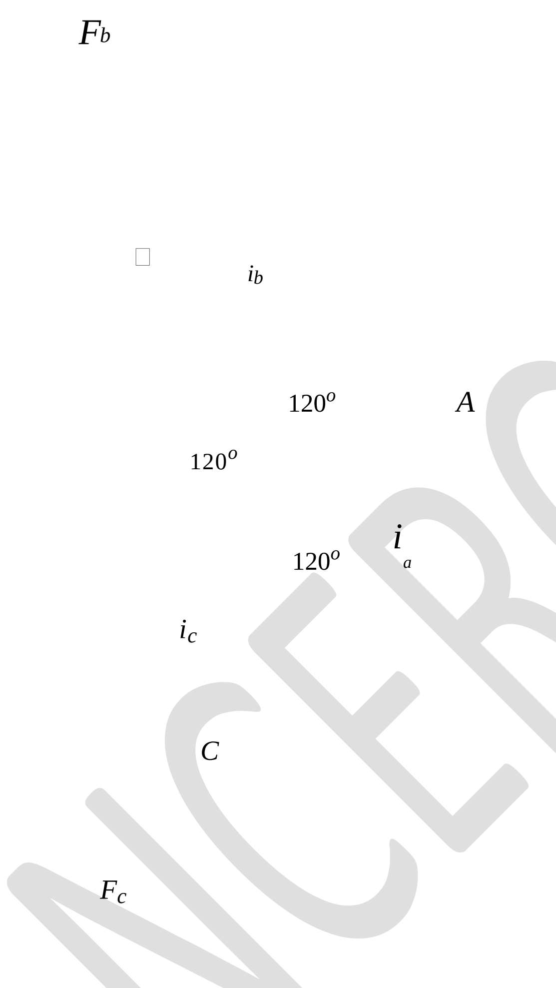

Figure 2: A symmetrical 2 pole 3phase winding on the rotor

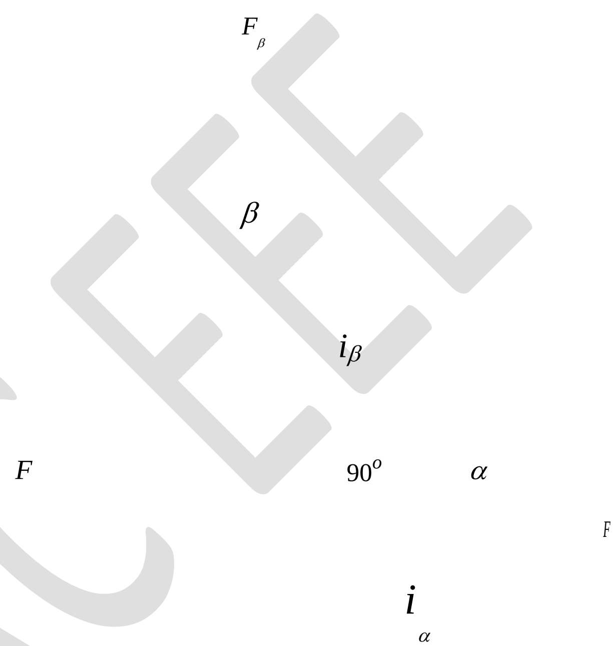

Figure 3: A balanced 2phase winding of the rotor

Maximum values of mmfs F_a , F_b , F_c are shown along their respective phase axes. The combined effect of these three mmfs results in mmf of constant magnitude rotating at a constant angular velocity depending on the poles and frequency. If the three currents in the rotor are:

$$i_a = I_m \cos(\omega t);\ i_b = I_m \cos\left(\omega t - \frac{2\pi}{3}\right);\ i_c = I_m \cos\left(\omega t - \frac{4\pi}{3}\right) \quad (22)$$

$$\left(\quad 3 \right) \qquad \left(\quad 3 \right)$$

The currents given in **equation 22** produce a mmf of constant magnitude $\frac{3I_m N_r}{2}$ rotating with respect to three phase winding at the frequency of ω. In **Figure 3**, a balanced two phase winding is represented by two orthogonal coils α , β on the rotor. For the sake of convenience the axes of phase $\boldsymbol{A}$ and α are taken to be coincident. The two phase currents flowing in the winding is given by

$$i_\alpha = I_m \cos(\omega t); \; i_\beta = I_m \cos\left(\omega t - \frac{\pi}{2}\right) \tag{23}$$

These two phase currents result in a mmf of constant magnitude $I_m N_r$ rotating with respect to the two phase windings at the frequency of the currents. The mmf of three phase and two phase systems can be rendered equal in magnitude by making any one of the following changes:

- By changing the magnitude of the two phase currents
- By changing the number of turns of the two phase windings
- By changing both the magnitude of currents and number of turns

In the following subsections each of the three cases are discussed.

Changing the magnitude of two phase currents

In this case the number of turns in the two phase winding is N_r which is same as that of the three phase windings. Hence, in order to have equal mmf, the new magnitude of the current in the two phases must be determined. To obtain the new values of the two phase currents the instantaneous three phase mmfs are resolved along the α axis shown in **Figure 3**:

$$i_\alpha N_r = \left(i_a \cos 0 + i_b \cos\frac{2\pi}{3} + i_c \cos\frac{4\pi}{3}\right) N_r \Rightarrow i_\alpha = i_a - \frac{1}{2}\left(i_b + i_c\right) \qquad (24)$$

Similarly, the resolving the three phase currents along the β axis gives

$$i_\beta N_r = \left(i_a \sin 0 + i_b \sin\frac{2\pi}{3} + i_c \sin\frac{4\pi}{3}\right) N_r \Rightarrow i_\beta = \left(\frac{\sqrt{3}}{2} i_b - \frac{\sqrt{3}}{2} i_c\right) \qquad (25)$$

For a balanced three phase system the sum of three currents is zero, that is

$$i_a + i_b + i_c = 0 \qquad (26)$$

Using **equation 26** into **equation 24** gives

$$i_\alpha = \frac{3}{2} i_a \tag{27}$$

Substituting the values of i_a, i_b and i_c from **equation 22** into **equations 25** and **27** gives

$$i_\alpha = \frac{3}{2} I_m \cos(\omega t); \; i_\beta = \frac{3}{2} I_m \sin(\omega t) \tag{28}$$

From **equation28** it can be seen that the magnitude of the two phase currents is 3/2 times the magnitude of the three phase currents. Since the number of turns per phase is same in both the three and two phase windings, the magnitude of phase e.m.fs of the two and three phase windings would be equal. The power per phase of the two phase system is $3/2VI_m$ and the power per phase of a three phase winding is VI_m . However, the total power produced by a two phase system is ($= 2 \cdot 3/2 \cdot VI_m = 3VI_m$) and that produced by a three phase system is $3VI_m$. Thus, the linear transformation is power invariant. The only disadvantage is that the transformation of current and voltage will differ because of presence of factor 3/2 in the current transformation. As factor 3/2 appears in current transformation and not in voltage transformation, the per phase parameters of the two phase and three phase machine will not be the same.

Changing the number of turns of two phase winding

If the number of turns of two phase winding is made 3/2 times that of the three phase winding, then for equal mmfs the following relation between the two phase and three phase currents holds:

$$\frac{3}{2} i_\alpha N_r = \left(i_a \cos 0 + i_b \cos\frac{2\pi}{3} + i_c \cos\frac{4\pi}{3} \right) N_r \Rightarrow \frac{3}{2} i_\alpha = i_a - \frac{1}{2}\left(i_b + i_c\right) = \frac{3}{2} i_a \Rightarrow i_\alpha = i_a \qquad (29)$$

$$\frac{3}{2} i_\beta N_r = \left(i_a \sin 0 + i_b \sin\frac{2\pi}{3} + i_c \sin\frac{4\pi}{3} \right) N_r \Rightarrow i_\beta = \frac{2}{3}\left(\frac{\sqrt{3}}{2} i_b - \frac{\sqrt{3}}{2} i_c \right) \qquad (30)$$

Substituting the values of i_a , i_b and i_c from **equation 22** into **equations 29** and **30** gives

$$i_\alpha = I_m \cos\left(\omega t\right); i_\beta = I_m \sin\left(\omega t\right) \qquad (31)$$

Since, the number of turns in the two phase winding is 3/2 times that of three phase winding, the per phase voltage of the two phase machine will be 3/2 times the per phase voltage of the three phase systems. Hence, The power per phase in two phase system= $\frac{3}{2} VI_m$

Total in two phase system= $3VI_m$

The power per phase in three phase system= VI_m

Total in three phase system= $3VI_m$

Here again the power invariance is obtained, but, as in the previous case, the transformation of current and voltage will differ because of the factor 3/2 in the voltage transformation. In this case the per phase parameters of the machine will be different for two and three phase systems.

Changing both the number of turns and magnitude of current of two phase winding

In this case both the magnitude of currents and number of turns of the two phase system are changed to obtain identical transformation for voltage and current. To do so the number of turns in the two phase winding is made $\sqrt{\frac{3}{2}}$ times that of three phase winding. Then for equal m.m.f the following holds

$$\sqrt{\frac{3}{2}}\, i_\alpha N_r = \left(i_a \cos 0 + i_b \cos\frac{2\pi}{3} + i_c \cos\frac{4\pi}{3} \right) N_r$$

$$\Rightarrow i_\alpha = \sqrt{\frac{2}{3}}\left(i_a - \frac{1}{2} i_b - \frac{1}{2} i_c \right) \Rightarrow i_\alpha = \sqrt{\frac{3}{2}}\, I_m \cos(\omega t)$$

(32)

$$\sqrt{\frac{3}{2}}\, i_\beta N_r = \left(i_a \sin 0 + i_b \sin\frac{2\pi}{3} + i_c \sin\frac{4\pi}{3} \right) N_r$$

- $$i_\beta = \sqrt{\frac{2}{3}}\left(0 + \frac{\sqrt{3}}{2} i_b - \frac{\sqrt{3}}{2} i_c \right) \Rightarrow i_\beta = \sqrt{\frac{3}{2}}\, I_m \sin(\omega t)$$ (33)

Since the number of turns in the two phase winding is $\sqrt{\frac{3}{2}}$ times that of three phase winding, the voltage per phase of the two phase winding is $\sqrt{\frac{3}{2}}$ times that of the three phase winding. Hence, the phase voltage and current of the two phase system are $\sqrt{\frac{3}{2}}$ times that of three phase system. This results in identical transformations for both the voltage and current and the per phase quantities of the machine, such as the impedance per phase, will be same for two and three phase systems.

Hence, the transformation equations for converting three phase currents into two phase currents, given by **equations 32** and **33**, can be expressed in matrix form as

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \quad (34)$$

The transformation matrix in **equation 34** is singular and hence i_a , i_b and i_c cannot be obtained from i_α , i_β . The matrix can be made square matrix if a third equation of constraint between i_a , i_b and i_c is introduced. Since, the magnitude and direction of the mmf produced by two and three phase systems are identical, the third current in terms of i_a , i_b and i_c should not produce any resultant air gap mmf. Hence, a zero sequence current is introduced and it is given by

$$i_0 = \frac{1}{3}\left(i_a + i_b + i_c\right) \qquad (35)$$

Due to the fact that sum of three phase currents in a balanced system is zero (**equation 26**), the zero sequence current does not produce any rotating mmf. Using the **equation 35** the matrix representation given in **equation 34** can be written as

$$\begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \qquad (36)$$

The transformation matrix now is non-singular and its inverse can be easily obtained.

Transformation from Rotating Axes ($\alpha, \beta, 0$) to Stationary Axes ($d, q, 0$)

In **Figure 3**, three phase and two phase windings are shown on the rotor and hence both $d, q, 0$ the windings rotate at the same speed and in the same direction. Hence, both the two phase and three phase windings are at rest with respect to each other.

In this section the transformation of rotating $\alpha, \beta, 0$ quantities to stationary

quantities is carried out. When transformation is carried out from rotating to stationary axes, the relative position of rotating axes varies with respect to stationary or fixed axes. Hence, the transformation matrix must have coefficients that are functions of the relative position of the moving α , β and fixed d , q axes. In **Figure 4** the rotating α , β axes are shown inside the circle and the stationary d , q axes are shown outside. The angle θ_r shown in **Figure 4** is such that at time $t = 0$, $\theta_r = 0$, that is, the α , β axis is aligned with the d , q axis.

F_q

Q

i_q β

θ i_β D

F_d

i_d

i_α α

Figure 4: The *d* and *q* axis on the rotor

At any time t , $\theta_r = \omega_r t$, where ω_r is the angular speed of the rotor. Assuming same d , number of turns in the α , β and q windings, the mmfs F_α and F_β can be resolved along the d , q axis as

$$F_d = F_\alpha \cos\theta_r + F_\beta \sin\theta_r \Rightarrow N_r i_d = N_r i_\alpha \cos\theta_r + N_r i_\beta \sin\theta_r$$

$$\Rightarrow i_d = i_\alpha \cos\theta_r + i_\beta \sin\theta_r \tag{37}$$

$$i_q = -i_\alpha \sin\theta_r + i_\beta \cos\theta_r$$

The **equation** 37 can be expressed in the matrix form as

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} \cos\theta_r & \sin\theta_r \\ -\sin\theta_r & \cos\theta_r \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix}$$

and (38)

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \begin{bmatrix} \cos\theta_r & -\sin\theta_r \\ \sin\theta_r & \cos\theta_r \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix}$$

Let the currents in d , q axis winding be

$$i_d = I_m \sin(\omega t + \phi)$$

$$i_q = I_m \cos(\omega t + \phi)$$

(39)

where

- is a constant arbitrary phase angle

Using **equation 38** and **equation 39**, the currents i_α , i_β are obtained as

$$i_\alpha = I_m \sin(\theta - \theta_r + \phi)$$

$$i_\beta = I_m \cos(\theta - \theta_r + \phi)$$

- $i_\alpha = I_m \sin(\omega t - \omega_r t + \phi)(\omega t - \omega_r t + \phi) i_\beta = I_m \cos$

(40)

where

$\varpi\iota. = \omega t$

In case the frequency of the d and q axis current is same as the speed of rotation of the rotor, then

$$i_\alpha = I_m \sin(\phi)$$

(41)

$$i_\beta = I_m \cos(\phi)$$

Thus, time varying currents in stationary d , q axis result in mmf which is identical to the mmf produced by constant currents (or d.c.) in rotating α , β axis.

In the above transformation the zero sequence current is not transformed and it can be taken into account by an additional column in the **equation 38**.

$$\begin{bmatrix} i_d \\ i_q \\ i_o \end{bmatrix} = \begin{bmatrix} \cos\theta_r & \sin\theta_r & 0 \\ -\sin\theta_r & \cos\theta_r & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \\ i_o \end{bmatrix} \tag{42}$$

Substituting the values of i_α, i_β, i_0 from **equation 32** into **equation 42** gives

$$\begin{bmatrix} i_d \\ i_q \\ i_o \end{bmatrix} = \frac{2}{3}\begin{bmatrix} \cos\theta_r & \cos\left(\theta_r - \frac{2\pi}{3}\right) & \cos\left(\theta_r + \frac{2\pi}{3}\right) \\ -\sin\theta_r & -\sin\left(\theta_r - \frac{2\pi}{3}\right) & -\sin\left(\theta_r + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

$$\left(i_d \;\; i_q \;\; i_0\right)^T = K_s \left(i_a \;\; i_b \;\; i_c\right)^T \qquad (43)$$

where

$$K_s = \frac{2}{3}\begin{bmatrix} \cos\theta_r & \cos\left(\theta_r - \frac{2\pi}{3}\right) & \cos\left(\theta_r + \frac{2\pi}{3}\right) \\ \sin\theta_r & \sin\left(\theta_r - \frac{2\pi}{3}\right) & \sin\left(\theta_r + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

The inverse transformation matrix is given by

$$\begin{bmatrix} \cos\theta & -\sin\theta_r & 1 \end{bmatrix}$$

$$K_S^{-1} = \frac{2}{3}\begin{bmatrix} \cos\theta_r & -\sin\theta_r & \frac{1}{2} \\ \cos\left(\theta_r - \frac{2\pi}{3}\right) & -\sin\left(\theta_r - \frac{2\pi}{3}\right) & \frac{1}{2} \\ \cos\left(\theta_r + \frac{2\pi}{3}\right) & -\sin\left(\theta_r + \frac{2\pi}{3}\right) & \frac{1}{2} \end{bmatrix} \tag{44}$$

The above transformation is valid for any electrical quantity such as current, voltage, flux linkage, etc. In general the three phase voltages, currents and fluxes can be converted into *dqo* phases using the following transformation matrices

$$\left(f_{qdo}\right)^T = K_s \left(f_{abc}\right)^T \tag{45}$$

where

$$\left(f_{qdo}\right)^T = \begin{bmatrix} f_q & f_d & f_o \end{bmatrix}; \quad \left(f_{abc}\right)^T = \begin{bmatrix} f_a & f_b & f_c \end{bmatrix}$$

Transformation of Induction Motor Quantities into Stationary d, q Axis

The three phase stator and rotor wind axis are shown in **Figure 5a**. In **Figure 5a**, the subscript ***s*** represents stator quantities and ***r*** represents rotor quantities. For the stator

δ, β axis and d , q axis are coincident as shown in **Figure 5b** and hence there is no difference between α , β and d , q stator quantities. Examination of **Figure 5a** and **5b** reveals that phase A_s coincides with the phase α axis or phase d axis of the 2 phase

machines. As a result of this, the results obtained for d axis quantities apply equally well to the α phase of the 2 phase machine. The conversion of 3 phase stator winding to 2 phase stator winding is given by **equation 36** as

$$\begin{bmatrix} i_{\alpha s} \\ i_{\beta s} \\ i_{0s} \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} \qquad (46)$$

Since the α , β and d , q axis both lie on the stator are stationary with respect to each other, the transformation from α , β to d , q axis is given by

$$\begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} i_{\alpha s} \\ i_{\beta s} \end{bmatrix} \qquad (47)$$

[10] s

β_r

α_s

α_r

Figure 5a: Equivalent 2 phase induction machine in α, β axis

Q_s

Q_r

D_r D_s

Figure 5b: Equivalent 2 phase induction machine in d , q axis

In case of rotor currents, the transformation from 3 phase to 2 phase is given by

$\lceil \quad r \quad | \quad \lfloor i_{0r}$

$i_{\alpha} \quad | \; i \quad {}_{\beta r}$

$$\begin{bmatrix} i_{\alpha r} \\ i_{\beta r} \\ i_{0r} \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} i_{ar} \\ i_{br} \\ i_{cr} \end{bmatrix} \qquad (48)$$

The transformation 2 phase rotor currents from rotating α, β to stationary d, q axis is given by

$$\begin{bmatrix} i_{dr} \\ i_{qr} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} i_{\alpha r} \\ i_{\beta r} \end{bmatrix} \qquad (49)$$

The rotor voltages are related by identical expressions. The frequency of the currents in 3phase rotor windings or in 2phase rotor windings is the slip frequency sf. Since the d, q axes windings are stationary, the frequency of rotor currents i_{dr} and i_{qr} is the line frequency f.

The Machine Performance Equations

The general voltage equation for the machine shown in **Figure 5b** is

$$\begin{bmatrix} v_{ds} \\ v_{qs} \\ v_{dr} \\ v_{qr} \end{bmatrix} = \begin{bmatrix} r_{ds} + L_{ds}p & 0 & M_d p & 0 \\ 0 & r_{qs} + L_{qs}p & 0 & M_q p \\ M_d p & -M_q\omega_r & r_{dr} + L_{dr}p & -L_{qr}\omega_r \\ M_d\omega_r & M_d p & L_{dr}\omega_r & r_{qr} + L_{qr}p \end{bmatrix} \begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr} \end{bmatrix} \tag{50}$$

where $p = \frac{d}{dt}$

The following points about the IM are to be considered:

- Stator and rotor both have balanced winding configurations, hence: $r_{ds} = r_{qs} = r_s$ =resistance of each stator coil

  $r_{dr} = r_{qr} = r_r$ =resistance of each rotor coil

- Since the air gap is uniform, the self inductances of d and q axis of the stator winding are equal and that of the rotor windings are also equal, that is

  $L_{ds} = L_{qs} = L_s$ =self inductance of the stator winding

  $L_{dr} = L_{qr} = L_r$ =self inductance of the rotor winding

- The d and q axis coils are identical, the mutual inductance between the stator and rotor d axis coils is equal to the mutual inductance between stator and rotor q axis coils, that is

  $M_d = M_q = L_m$

In an induction machine the rotor windings are short circuited, therefore no emf exists in the winding of the rotor and $v_{dr} = v_{qr} = 0$. Since the rotor winding is short circuited, the direction of i_{dr} and i_{qr} are reversed and this has to be taken into account in the general voltage equations.

Based on the above discussion, the general voltage equation becomes:

$$\begin{bmatrix} v_{ds} \\ v_{qs} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r_s + L_s p & 0 & -L_m p & 0 \\ 0 & r_s + L_s p & 0 & -L_m p \\ L_m p & -L_m \omega_r & -(r_r + L_r p) & L_r \omega_r \\ L_m \omega_r & L_m p & -L_r \omega_r & -(r_r + L_r p) \end{bmatrix} \begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr} \end{bmatrix} \tag{51}$$

The voltage equations given in **equation 51** can be written as:

$$V = [R]I + [L]pI + [G]\omega_r I$$ where

$$V = \begin{bmatrix} v_{ds} \\ v_{qs} \\ v_{dr} \\ v_{qr} \end{bmatrix}; \quad I = \begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr} \end{bmatrix}; \quad [R] = \begin{bmatrix} r_s & 0 & 0 & 0 \\ 0 & r_s & 0 & 0 \\ 0 & 0 & r_s & 0 \\ 0 & 0 & 0 & r_s \end{bmatrix} \tag{52}$$

$$[L] = \begin{bmatrix} L_s & 0 & L_m & 0 \\ 0 & L_s & 0 & L_m \\ L_m & 0 & L_r & 0 \\ 0 & L_m & 0 & L_r \end{bmatrix}; \quad [G] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -L_m & 0 & L_r \\ L_m & 0 & -L_r & 0 \end{bmatrix}$$

The instantaneous input power to the machine is given by

$$P = I^T V = I^T[R]I + I^T[L]pI + I^T[G]\omega_r I \tag{53}$$

In **equation 53** the term $I^T[R]I$ represents the stator and rotor resistive losses. The term $I^T[L]pI$ denotes the rate of change of stored magnetic energy. Hence, what is left of the power component must be equal to the air gap power given by the term $I^T[G]\omega_r I$. The air gap torque is given by

$$\omega_m T_e = I^T[G]\omega_r I$$

$$\omega_r = \frac{N_p}{2}\omega_m$$

• T_e

$$= \frac{N_p}{2} I^T [G] I \qquad (54)$$

where

ω_m is the mechanical speed of the rotor

and N_p is the number of poles

Substituting the value of $[G]$ from **equation52** into **equation 54** gives

$$T_e = \frac{N_p}{2} L_m \left(i_{qs} i_{dr} - i_{ds} i_{qr} \right) \qquad (55)$$

The torque given by **equation 55** can also be written as

$$T_e = \frac{N_p}{2} L_m \left(\lambda_{qr} i_{dr} - \lambda_{dr} i_{qr} \right) \qquad (56)$$

.

.

References:

2 R. Krishnan, *Electric motor drives: modeling, analysis, and control*, Prentice Hall, 2001

3 P. C. Krause, O. Wasynczuk, S. D. Sudhoff, *Analysis of electric machinery*, IEEE Press, 1995

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Lecture 21: Switch reluctance motors, their configurations and optimization

Field Oriented Control of Induction Motor

Introduction

The topics covered in this chapter are as follows:

- ⇒ Field Oriented Control (FOC)
- ⇒ Direct Rotor Oriented FOC
- ⇒ Indirect Rotor Oriented FOC

Field Oriented Control (FOC)

In an Electric Vehicle, it is required that the traction motor is able to deliver the required torque almost instantaneously. In an induction motor (IM) drive, such performance can be achieved using a class of algorithms known as ***Field Oriented Control*** *(FOC)*. There are varieties of FOC such as:

- ⇒ Stator flux oriented
- ⇒ Rotor flux oriented
- ⇒ Air gap flux oriented

Each of the above mentioned control method can be implemented using ***direct*** or ***indirect*** methods.

The basic premise of FOC may be understood by considering the current loop in a uniform magnetic field as shown in **Figure 1a**. From Lorenz force equation, it can be seen that the torque acting on the current loop is given by

$$T_e = -2\,BiNLr\sin\theta$$

where

B is the flux density

i is the current (1)

□ is the number of turns
L is the length of the coil
r is the radius of the coil

i^s_{qdr} λ^s_{qdr}

Current vector

r

θ

B

θ

r

q axis

d axis

Figure 1a: A coil in a magnetic field **Figure 1b: Orientation of magnetic field in IM**

From **equation 1** it is evident that the torque is maximised when the current vector is perpendicular to the magnetic field. The same conclusion can be applied to an IM. In **Figure 1b** orientations of magnetic fields and currents in an IM are shown. The rotor current and flux linkage vectors are shown in **Figure 1** at some instant of time. Hence, the torque produced by the motor (refer to Lecture 19) is given by

$$T_e = \frac{3}{2}\frac{P}{2}\left(\lambda'_{qr} i'_{dr} - \lambda'_{dr} i'_{qr}\right) \qquad (2)$$

The **equation 2** can be re-written as

$$T_e = -\frac{3}{2}\frac{P}{2}\left|\lambda'_{qdr}\right|\left|i'_{qdr}\right|\sin\theta \qquad (3)$$

The **equation 3** is analogous to **equation 1**. Hence, for a given magnitude of flux linkage, torque is maximised when the flux linkage and current vectors are perpendicular. Therefore, it is desirable to keep the rotor flux linkage perpendicular to rotor current vector.

In the analysis of FOC the following convention will be used:

- The parameters with a superscript " s " are in stator frame of reference.
- The parameters with a superscript " e " are in synchronous frame of reference.
- The parameters with subscript " r " indicate rotor parameters.
- The parameters with subscript " s " indicate stator parameters.
- All rotor quantities are referred to stator using the turns ratio of the windings (Lecture 17) and hence " ' " is dropped.

In case of singly excited IMs (*in singly excited IM, the rotor winding is not fed by any external voltage source. In case of wound rotor machines, they are short circuited using slip rings. For cage IMs, the rotor bars are short circuited at the terminals*), the rotor flux linkage vector and rotor current vector are always perpendicular. The voltage equations for the IM (refer to Lecture 19) in synchronous frame of reference are

$$
\begin{aligned}
v_{qs}^e &= r_s i_{qs}^e + \omega_e \lambda_{ds}^e + p\lambda_{qs}^e \\
v_{ds}^e &= r_s i_{ds}^e - \omega_e \lambda_{qs}^e + p\lambda_{ds}^e \\
v_{os}^e &= r_s i_{os}^e + p\lambda_{os}^e \\
v_{qr}^e &= r_r i_{qr}^e + (\omega_e - \omega_r)\lambda_{dr}^e + p\lambda_{qr}^e \\
v_{dr}^e &= r_r i_{dr}^e - (\omega_e - \omega_r)\lambda_{qr}^e + p\lambda_{dr}^e \\
v_{or}^e &= r_r i_{or}^e + p\lambda_{or}^e
\end{aligned}
\qquad (1)
$$

where

ω_e is the rotational speed of synchronous frame of reference

In case of singly excited IM, the rotor voltages are zero, that is $v_{qr}^e = 0$, $v_{dr}^e = 0$ and $v_{or}^e = 0$.

Hence, the rotor currents can be obtained as

$$0 = r_r i_{qr}^e + (\omega_e - \omega_r)\lambda_{dr}^e + p\lambda_{qr}^e \Rightarrow i_{qr}^e = -\frac{1}{r_r}(\omega_e - \omega_r)\lambda_{dr}^e - p\lambda_{qr}^e$$

$$0 = r_r i_{dr}^e - (\omega_e - \omega_r)\lambda_{qr}^e + p\lambda_{dr}^e \Rightarrow i_{dr}^e = \frac{1}{r_r}(\omega_e - \omega_r)\lambda_{qr}^e - p\lambda_{dr}^e \qquad (2)$$

$$0 = r_r i_{or}^e + p\lambda_{or}^e \Rightarrow i_{or}^e = -\frac{p\lambda_{or}^e}{r_r}$$

Since steady state operation of IM is considered, the time derivative term of flux linkage in **equation 2** will vanish. Hence, the rotor currents are:

$$i_{qr}^{e} = -\frac{1}{r_r}\left(\omega_e - \omega_r\right)\lambda_{dr}^{e}$$

$$i_{dr}^{e} = \frac{1}{r_r}\left(\omega_e - \omega_r\right)\lambda_{qr}^{e} \qquad (3)$$

$$i_{or}^{e} = 0$$

The dot product of the rotor flux linkage and rotor current vectors may be expressed as

$$\lambda_{qdr}^{e} \,.\, i_{qdr}^{e} = \lambda_{qr}^{e} \,.\, i_{qr}^{e} + \lambda_{dr}^{e} \,.\, i_{dr}^{e} \qquad (4)$$

For m **equation 5** it can be seen that the dot product between the rotor flux and rotor current vectors is zero in case of singly excited IM. Hence, it can be concluded that the rotor flux and rotor current vectors are perpendicular to each other in steady state operation. The defining feature of FOC is that this characteristic (that the rotor flux and rotor current vectors are perpendicular to each other) is maintained during transient conditions as well.

(7)

By suitable choice of θ_s on an instantaneous basis, **equation 6** can be achieved. Satisfying **equation 7** can be accomplished by forcing d -axis stator current to remain constant. To see this, consider the d -axis rotor voltage equation

$$0 = r_r\, i_{dr}^e + (\omega_e - \omega_r)\lambda_{qr}^e + p\lambda_{dr}^e$$

Since $\lambda_{qr}^e = 0$, **equation 8** can be written as

$$0 = r_r\, i_{dr}^e + p\lambda_{dr}^e$$

Substituting the values of i_{qr}^e and i_{dr}^e from **equation 3** into **equation 4** gives

$$\lambda_{qdr}^e \cdot i_{qdr}^e = -\frac{\lambda_{qr}^e}{r_r}(\omega_e - \omega_r)\lambda_{dr}^e + \frac{\lambda_{dr}^e}{r_r}(\omega_e - \omega_r)\lambda_{qr}^e = 0 \qquad (6)$$

In both direct and indirect FOC, the 90^o vector can be achieved in two steps:

- The first step is to ensure that

$$\lambda_{qr}^e = 0$$

- The second s

The d - axis roto r

flux linkage is given by (refer Lecture 19):

$$\lambda_{dr}^{e} = L_{lr}\, i_{dr}^{e} + L_m \left(i_{ds}^{e} + i_{dR}^{e} \right)$$

Substituting the value of λ_{dr}^{e} from **equation 10** into **equation 9** gives: (11)

$$pi_{dr}^{e} = -\frac{r_r}{L_{lr}} i_{dr}^{e} - \frac{L_m}{L_{lr}} pi_{ds}^{e}$$

If i_{ds}^{s} is held constant, then $pi_{ds}^{e} = 0$ and the solution of **equation 11** becomes

$$i_{dr}^{e} = Ce^{-\left(\frac{r_r'}{L_{lr}}\right)t} \qquad (12)$$

where

C is a constant of integration

It is evident from **equation 12** that the rotor current i_{dr}^e will decay to zero and stay at zero regardless of other transients that may be taking place. Hence, the torque (from **equation 2**) is given by

$$T_e = \frac{3}{2}\frac{P}{2}\lambda_{dr}^e i_{qr}^e \tag{13}$$

The q -axis rotor flux is given by (refer Lecture 19)

$$\lambda_{qr}^e = L_{lr} i_{qr}^e + L_m\left(i_{qs}^e + i_{qr}^e\right) \tag{14}$$

Since, $\lambda_{qr}^e = 0$, the q -axis rotor current is given by

$$i_{qr}^e = -\frac{L_m}{L_{lr}} i_{qs}^e \tag{15}$$

Combining **equations 13** and **15** gives

$$T_e = \frac{3}{2}\frac{P}{2}\frac{L_m}{L}\lambda_{dr}^e i_{qs}^e \tag{16}$$

The d -axis rotor flux is given by (refer Lecture 19)

$$\lambda_{dr}^e = L_{lr} i_{dr}^e + L_m\left(i_{ds}^e + i_{dr}^e\right) \tag{17}$$

The **equation 7** gives $i_{dr}^e = 0$, hence **equation 17** can be written as

$$\lambda_{dr}^e = L_m i_{ds}^e \tag{18}$$

.

Together, **equation 19** and **equation 21** suggest the generic rotor flux oriented control shown in **Figure 2**.

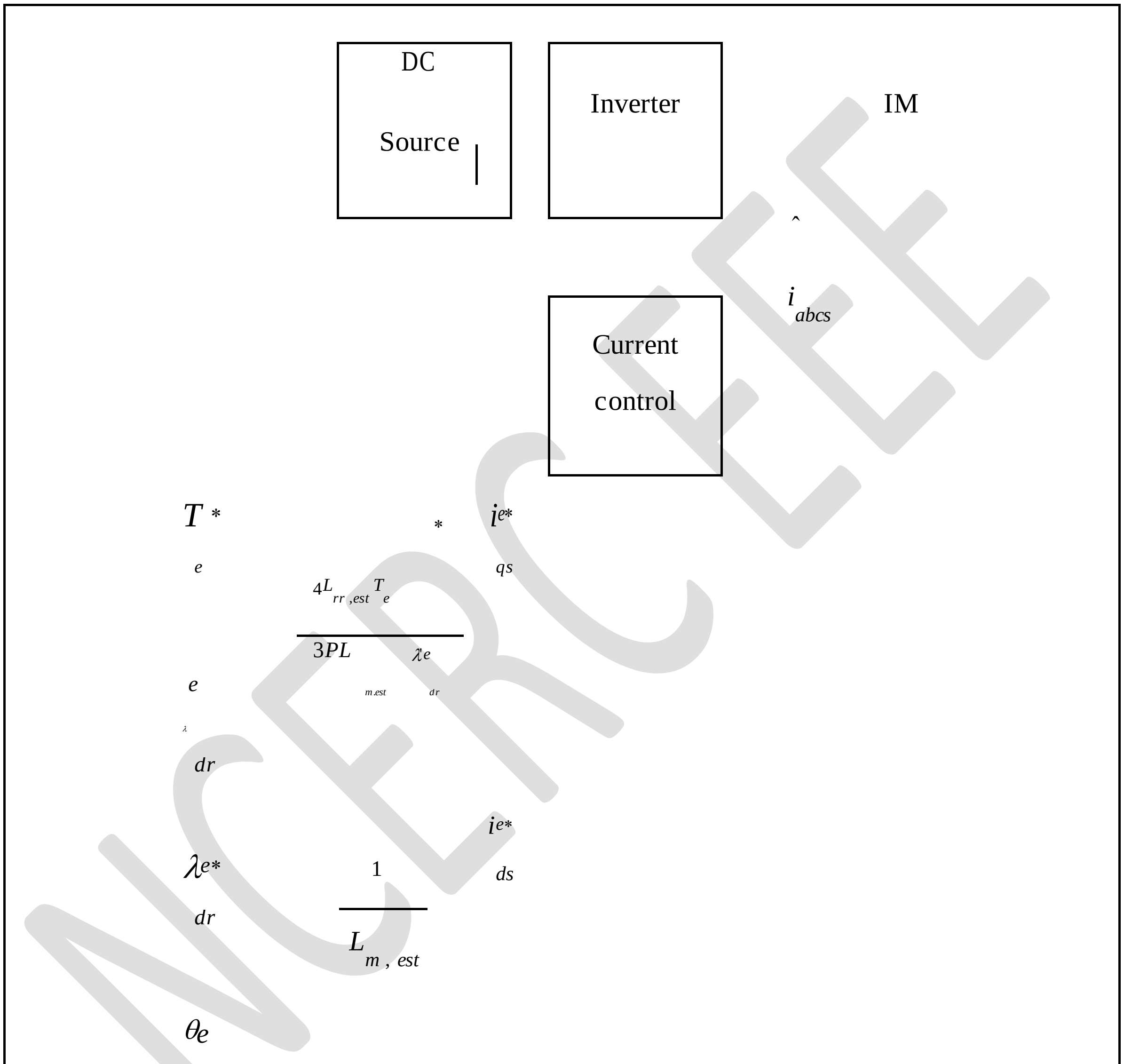


Figure 2: Generic Field Oriented Control (FOC) [1]

In **Figure 2** the variables of the form x^*, x and $\hat{x}$ denote ***command***, ***measured*** and ***estimated*** values respectively. In case of ***parameters that are estimated***, a subscript "**est**" is used. The working of the controller is as follows:

- Based on the torque command (T_e^*), the assumed values of the parameters and the estimated value of d -axis rotor flux $\hat{\lambda}_{dr^s}$ is used to formulate a q -axis stator current command $i_{qs}{}^{s*}$.

- The d -axis stator current command $i_{ds^s}{}^{*}$ is calculated such as to achieve a rotor flux command $\lambda_{dr}{}^{s*}$ (using **equation 12**).s
- The q -axis and d -axis stator current command is then achieved using a current source control.

The above description of rotor flux oriented FOC is incomplete with determination of $\hat{\lambda}_{dr^s}$ and θ_s . The difference between ***direct*** and ***indirect*** FOC is in how these two variables are determined.

Direct Rotor Oriented FOC

In direct FOC, the position of the synchronous reference frame (θ_e) is determined based on the values of q -axis and d -axis rotor flux linkages in the stationary reference frame. The relation of flux linkages in synchronous reference frame and stationary reference frame is

$$\begin{bmatrix} \lambda_{qr}^{e} \\ \lambda_{dr}^{e} \end{bmatrix} = \begin{bmatrix} \cos\theta_e & -\sin\theta_e \\ \sin\theta_e & \cos\theta_e \end{bmatrix} \begin{bmatrix} \lambda_{qr}^{s} \\ \lambda_{dr}^{s} \end{bmatrix} \qquad (19)$$

where

$\lambda_{qr}{}^{s}$ is the rotor q-axis flux linkage in stationary frame of reference

$\lambda_{dr}{}^{s}$ is the rotor d -axis flux linkage in stationary frame of reference

In order to achieve $\lambda_{qre} = 0$, it is sufficient to define the position of the synchronous reference frame as

$$\theta_e = \tan^{-1}\left(\frac{\lambda_{qr}^{s}}{\lambda_{dr}^{s}}\right) + \frac{\pi}{2} \qquad (20)$$

The difficulty with this approach is that λ_{qrs} and λ_{drs} are not directly measurable quantities.

However, they can be estimated using direct measurement of air gap flux. To measure the air gap flux, hall-effect sensors are placed in the air gap and used to measure the air-gap flux in q -axis and d -axis. Since the hall-effect sensors are stationary, the flux measured by them is in stationary reference frame. The flux measured by the sensors is the net flux in the air gap (combination of stator and rotor flux). The net flux in the air gap is given by:

$$\lambda_{qm}{}^{s} = L_m\left(i_{qs}{}^{s} + i_{qr}{}^{s}\right) \qquad (21)$$

where

L_m is the magnetization inductance

From **equation 21**, the rotor q-axis current is obtained as

$$i_{qr}^{s} = \frac{\lambda_{qm}^{s} - L_m i_{qs}^{s}}{L_m} \tag{22}$$

The q-axis rotor flux linkage is given by

$$\lambda_{qr}^{s} = L_{lr}\, i_{qr}^{s} + L_m \left(i_{qs}^{s} + i_{qr}^{s} \right) \tag{23}$$

Substituting the rotor q-axis current from **equation 22** into **equation 23** gives

$$\lambda_{qr}^{s} = \frac{L_{lr}}{L_m} \lambda_{qm}^{s} - L_{lr}\, i_{qs}^{s} \tag{24}$$

.

An identical derivation for d -axis gives

$$\lambda_{dr}^{s} = \frac{L_{r}}{L_{m}} \lambda_{dm}^{s} - L_{lr}\, i_{ds}^{s} \quad (25)$$

The implementation of this control strategy is shown in **Figure 3a** and **b**.

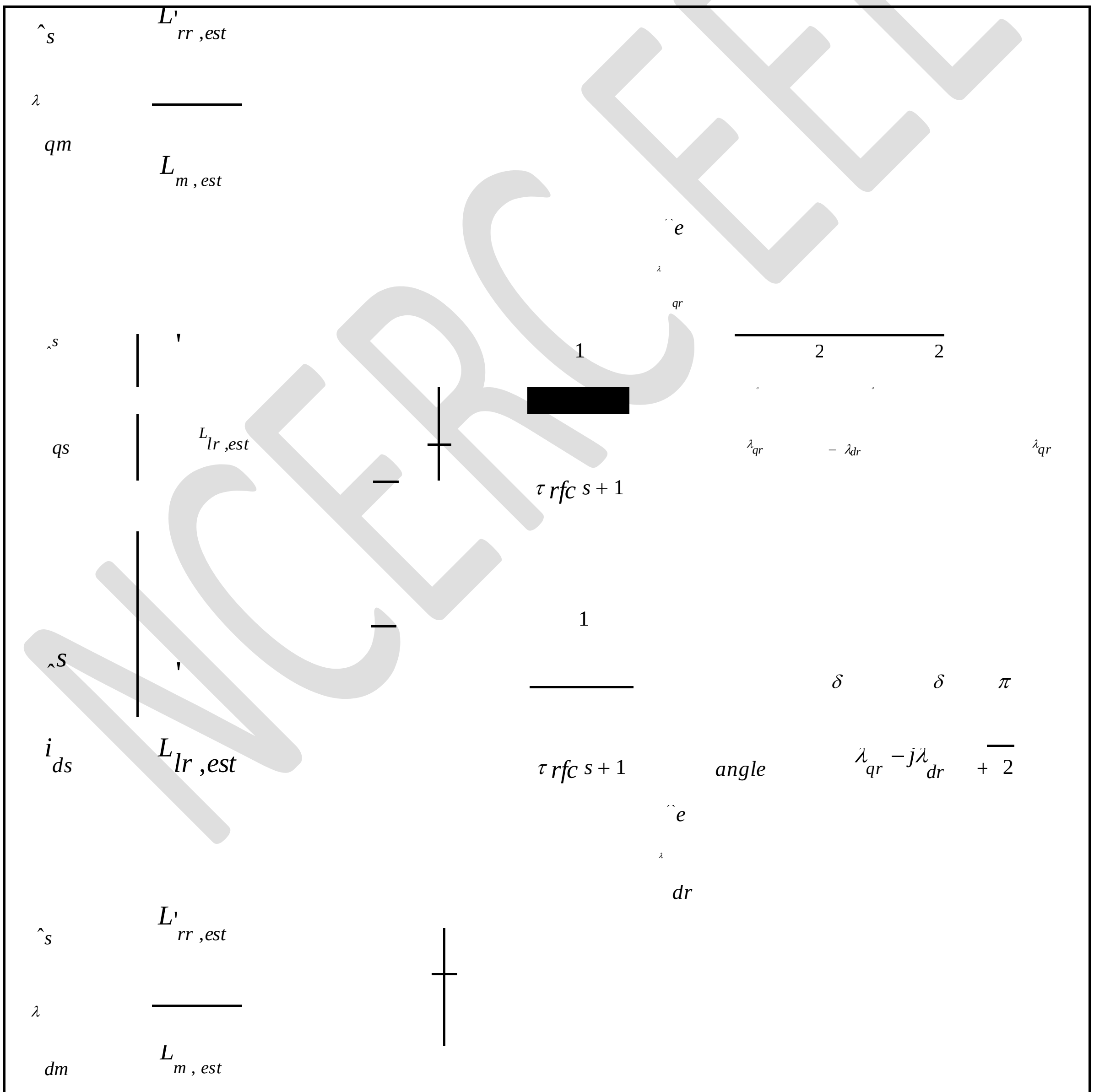


Figure 3a: Rotor flux calculation in Direct FOC [1]

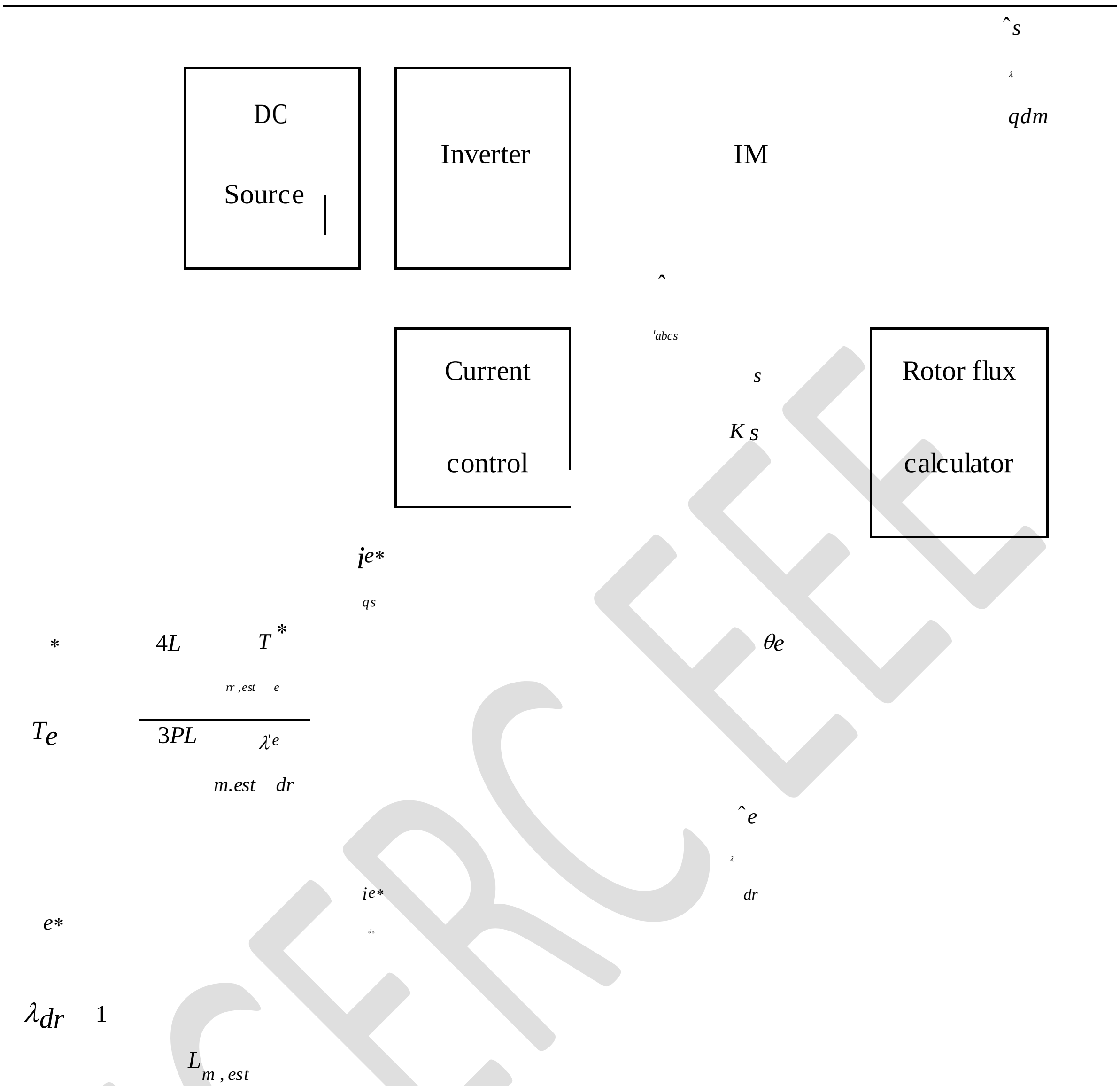


Figure 3b: Control strategy for Direct FOC [1]

Indirect Rotor Oriented FOC

The direct FOC is problematic and expensive due to use of hall-effect sensors. Hence, indirect FOC methods are gaining considerable interest. The indirect FOC methods are more sensitive to knowledge of the machine parameters but do not require direct sensing of the rotor flux linkages.

The q -axis rotor voltage equation in synchronous frame is

$$0 = r_r\, i_{qr}^{e} + (\omega_e - \omega_r)\lambda_{dr}^{e} + p\lambda_{qr}^{e} \tag{26}$$

Since $\lambda_{qr}^{e} = 0$ for direct field oriented control, **equation 26** becomes

$$0 = r_r\, i_{qr}^{e} + (\omega_e - \omega_r)\lambda_{dr}^{e} \tag{27}$$

$$\Rightarrow \omega_e = \omega_r - r_r \frac{i_{qr}^{e}}{\lambda_{dr}^{e}}$$

Substituting the values of i_{qre} and λ_{dr}^{e} from **equation 15** and **18** respectively into **equation**

□ gives

$$\omega_e = \omega_r + \frac{r_r}{L_{lr}}\frac{i_{qs}^{e}}{\lambda_{ds}^{e}} \tag{28}$$

From **equation 28** it can be observed that instead of establishing θ_e using the rotor flux as shown in **Figure 3**, it can be determined by integrating ω_e given by **equation 28** where ω_e is given as:

$$\omega = \omega + \frac{r}{L}\frac{i_{qs}^{e*}}{\lambda^{e*}} \tag{29}$$

er

lr ds

The **equation 29** does satisfy the conditions of FOC. In order to check it, consider the rotor voltage equations for the q -axis and d -axis:

$$0 = r_r\, i_{qr}{}^e + (\, \omega_e - \omega_r\,)\lambda_{dr}{}^e + p\lambda_{qr}{}^e \tag{30}$$

$$0 = r_r\, i_{dr}{}^e + (\, \omega_e - \omega_r\,)\lambda_{qr}{}^e + p\lambda_{dr}{}^e \tag{31}$$

Substituting ω_e from **equation 29** into **equations 30** and **31** gives

$$0 = r_r\, i_{qr}^{e} + \frac{r_r\, i_{qs}^{e*}}{L_{lr}\, i_{ds}^{e*}}\lambda_{dr}^{e} + p\lambda_{qr}^{e} \tag{32}$$

$$0 = r_r\, i_{dr}^{e} + \frac{r_r\, i_{qs}^{e*}}{L_{lr}\, i_{ds}^{e*}}\lambda_{qr}^{e} + p\lambda_{dr}^{e} \tag{33}$$

Substituting the value of d-axis rotor flux from **equations 17** into **equation 33** gives

$$0 = r_r \left(\frac{\lambda_{qr}^e - L_m i_{qs}^{e*}}{L_{lr}} \right) i_{qr}^e + \frac{r_r}{L_{lr}} \frac{i_{qs}^{e*}}{i_{ds}^{e*}} \left(L_{lr} i_{dr}^{e*} + L_m i_{ds}^{e*} \right) + p\lambda_{qr}^e \qquad (34)$$

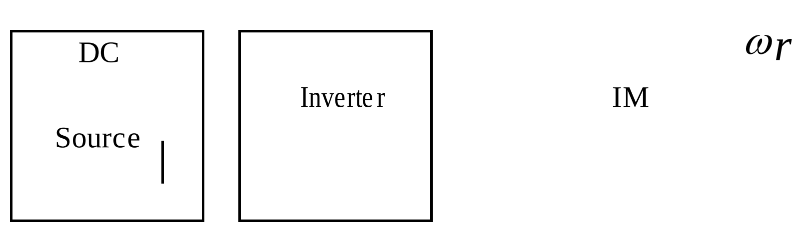
$$0 = r_r i_{dr}^e - \frac{r_r}{L_{lr}} \frac{i_{qs}^{e*}}{i_{ds}^{e*}} \lambda_{qr}^e + p \left(L_{lr} i_{qr}^e + L_m i_{ds}^{e*} \right) \qquad (35)$$

If the d-axis rotor current is held constant, then $pi_{dr}^{e*} = 0$ and rearranging **equations 34** and **35** gives

$$p\lambda_{qr}^e = -\frac{r_r}{L_{lr}} \lambda_{qr}^e - r_r \frac{i_{qs}^{e*}}{i_{ds}^{e*}} i_{dr}^e \qquad (36)$$

$$pi_{dr}^e = -\frac{r_r}{L_{lr}} i_{dr}^e + \frac{r_r i_{qs}^{e*}}{L_{lr} L_{lr} i_{ds}^{e*}} \lambda_{qr}^e \qquad (37)$$

The solution of **equations 37** and **38** will decay to zero (same argument as used for **equation 12**), hence λ_{qr}^e and i_{qr}^e will eventually become zero. In **Figure 4** the implementation of ***indirect FOC*** is shown and it is much simpler than the ***direct FOC***.



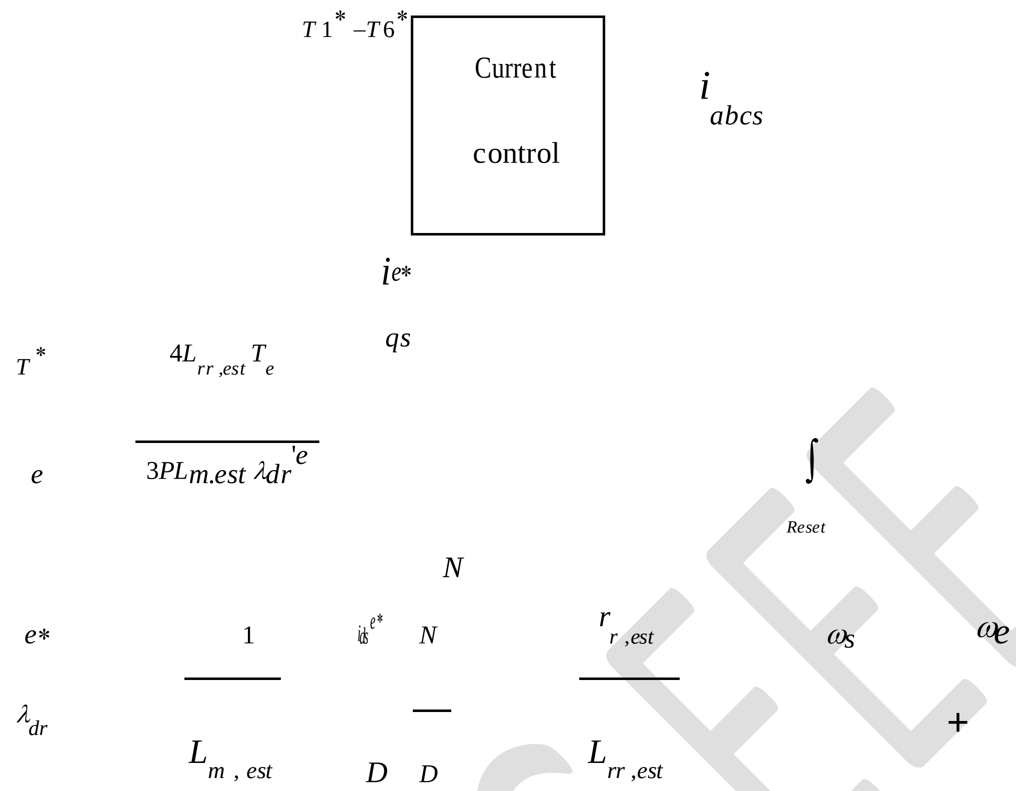


Figure 4: Indirect FOC [1]

References:

□ P. C. Krause, O. Wasynczuk, S. D. Sudhoff, *Analysis of electric machinery*, IEEE Press, 1995

Suggested Reading

□ R. Krishnan, *Electric motor drives: modeling, analysis, and control*, Prentice Hall, 2001

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Module 6: A.C. Electrical Machines for Hybrid and Electric Vehicles

Lecture 17: Induction motors, their configurations and optimization for HEV/EVs

Fundamentals of Electrical Machines

Introduction

The topics covered in this chapter are as follows:

ω. Electrical Machines in EVs and HEVs

ξ. Physical Concepts of Torque production

ψ. Why Should the Number of Poles on Stator Equal to the Number of Poles on Rotor

ζ. How Continuous Torque is Produced by a Motor

αα. Rotating Magnetic Field

ββ. How to Create the Second Magnetic Field

χχ. Electrical and Mechanical Angle

Electrical Machines in EVs and HEVs

Vehicle propulsion has specific requirements that distinguish stationary and onboard motors. Every kilogram onboard the vehicle represents an increase in structural load. This increase structural load results in lower efficiency due to increase in the friction that the vehicle has to overcome. Higher efficiency is equivalent to a reduction in energy demand and hence, reduced battery weight.

The fundamental requirement for traction motors used in EVs is to generate propulsion torque over a wide speed range. These motors have intrinsically neither nominal speed nor nominal power. The power rating mentioned in the catalog and on the name plate of the motor corresponds to the maximum power that the drive can deliver. Two most commonly used motors in EV propulsion are Permanent Magnet (PM) Motors and Induction Motors (IM). These two motors will be investigated in detail in the coming lectures. However, before going into the details of these machines some basic fundamentals of electrical machines, such as torque production, are discussed in this chapter.

Physical Concepts of Torque Production

In **Figure 1a** a stator with 2 poles and a cylindrical rotor with a coil are shown. When only the stator coils are energized, stator magnetic flux is set up as shown in **Figure 1a**. The magnetic field for case when only the rotor coil is energized is shown in **Figure 1b**. In case when both the stator and rotor coils are energized, the magnetic resultant magnetic field is shown in **Figure 1c**. Since in this case the magnetic flux lines behave like stretched band, the rotor conductor experiences a torque in the direction shown in **Figure 1c**. From **Figure 1c** it can be seen that the stator ***S*** pole attracts the rotor ***N*** pole and repels the rotor ***S*** pole, resulting in clockwise torque. Similarly stator ***N*** pole attracts rotor ***S*** pole and repels rotor ***N*** pole, resulting again in clockwise torque.

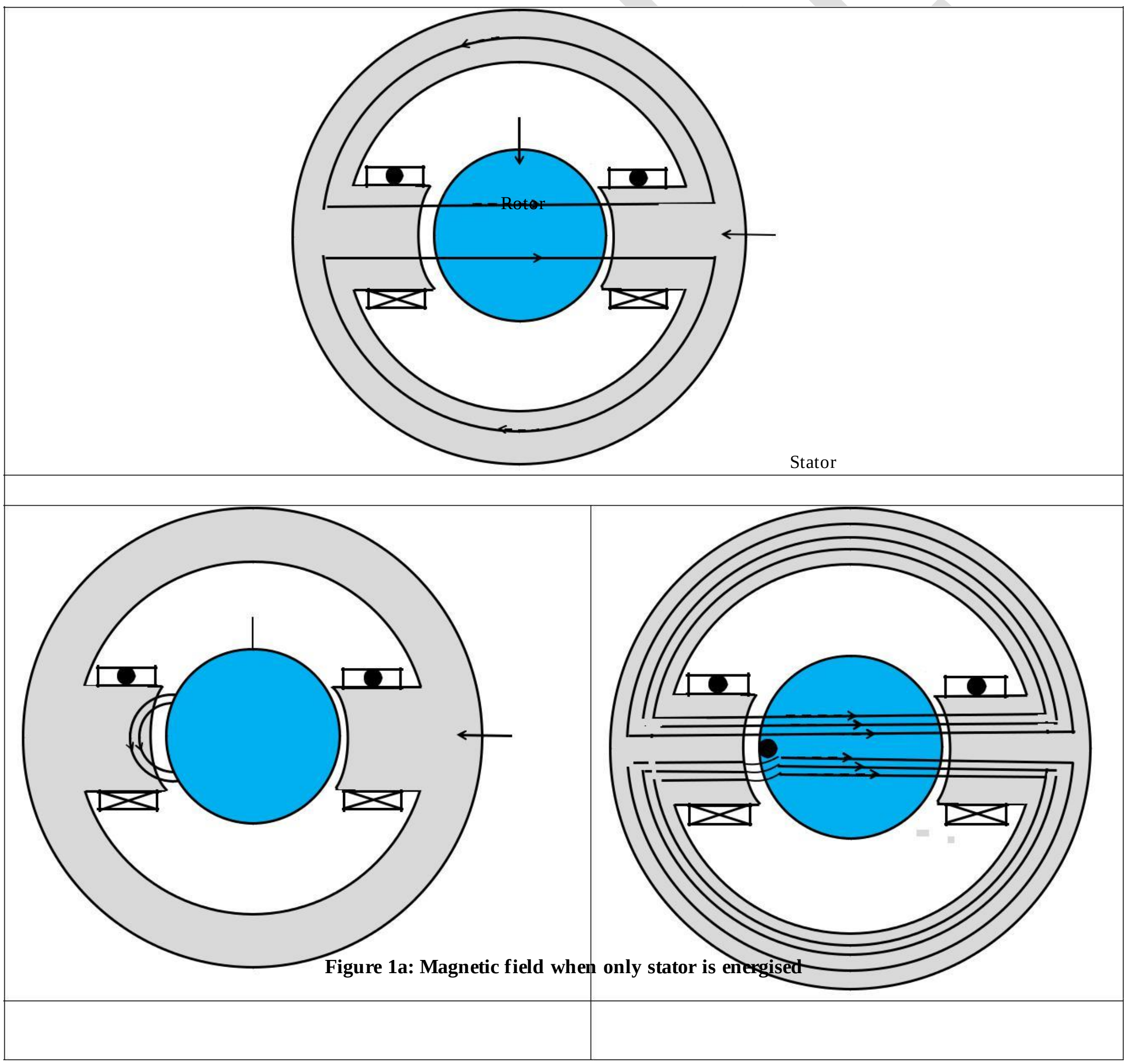


Figure 1a: Magnetic field when only stator is energised

Rotor

Stator

Figure 1b: Magnetic field when only rotor is energised

Figure 1c: Magnetic field when both stator and rotor are energised

The total torque is shown in **Figure 1c**. This torque is developed due to the interaction of stator and rotor magnetic fields and hence is known as ***interaction torque*** or ***electromagnetic torque***. The magnitude of the electromagnetic torque (T_{em}) or interaction torque is given by

$$T_{em} \propto (H_s)(H_r)\sin\delta \qquad (1)$$

where

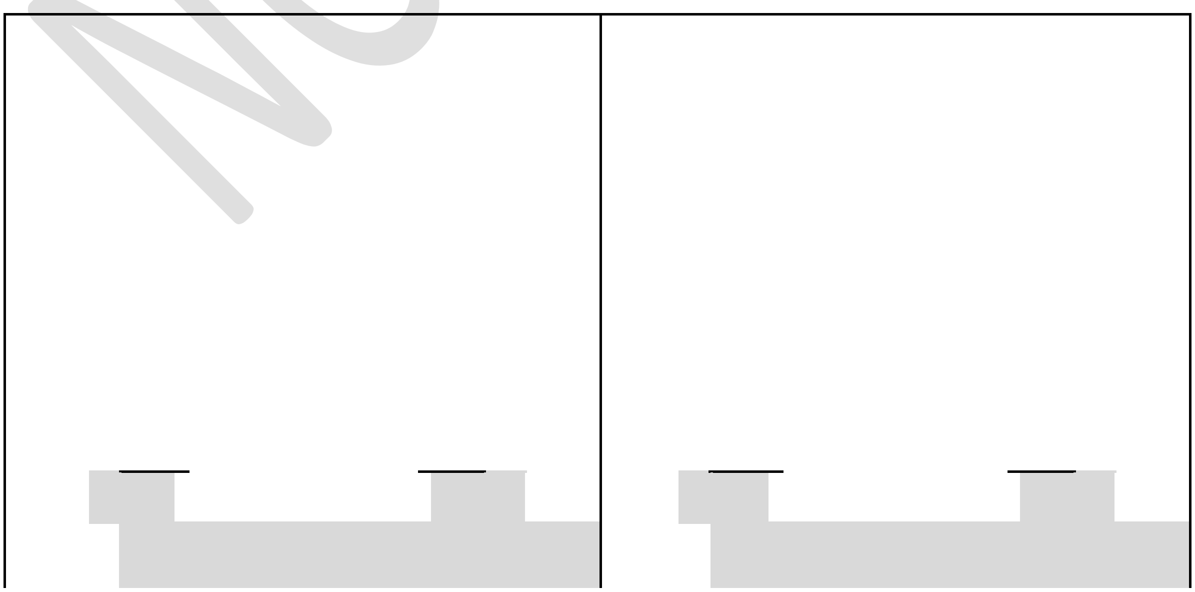
H_s is the magnetic field created by current in the stator winding

H_r is the magnetic field created by current in the rotor winding

• is the angle between stator and rotor magnetic field

Another configuration of the motor, with the flux lines, is shown in **Figure 2a**. Since the magnetic flux has a tendency to follow a minimum reluctance path or has a tendency to shorten its flux path, the rotor experiences an anti-clockwise torque. From **Figure 2a** it can be seen that the flux lines will have a tendency to align the rotor so that the reluctance encountered by them is reduced. The least reluctance position of the rotor is shown in

Figure 2b.



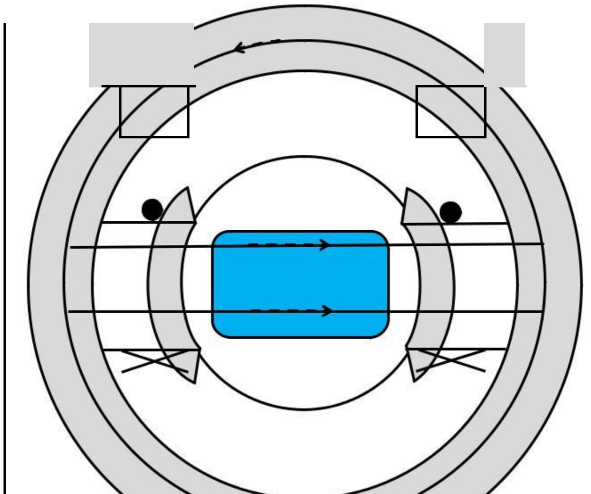


Figure 2a: Magnetic field when rotor axis is aligned with stator poles

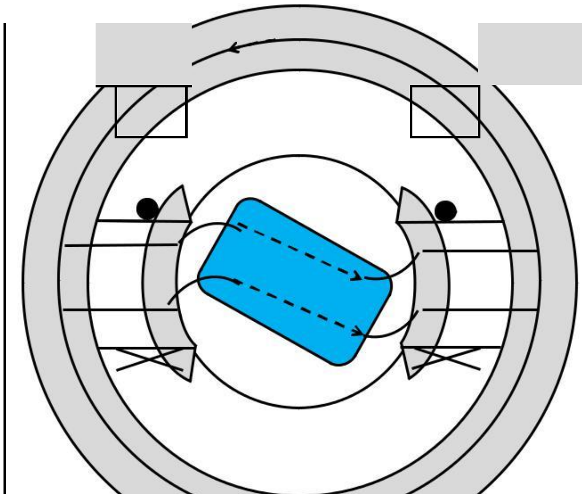


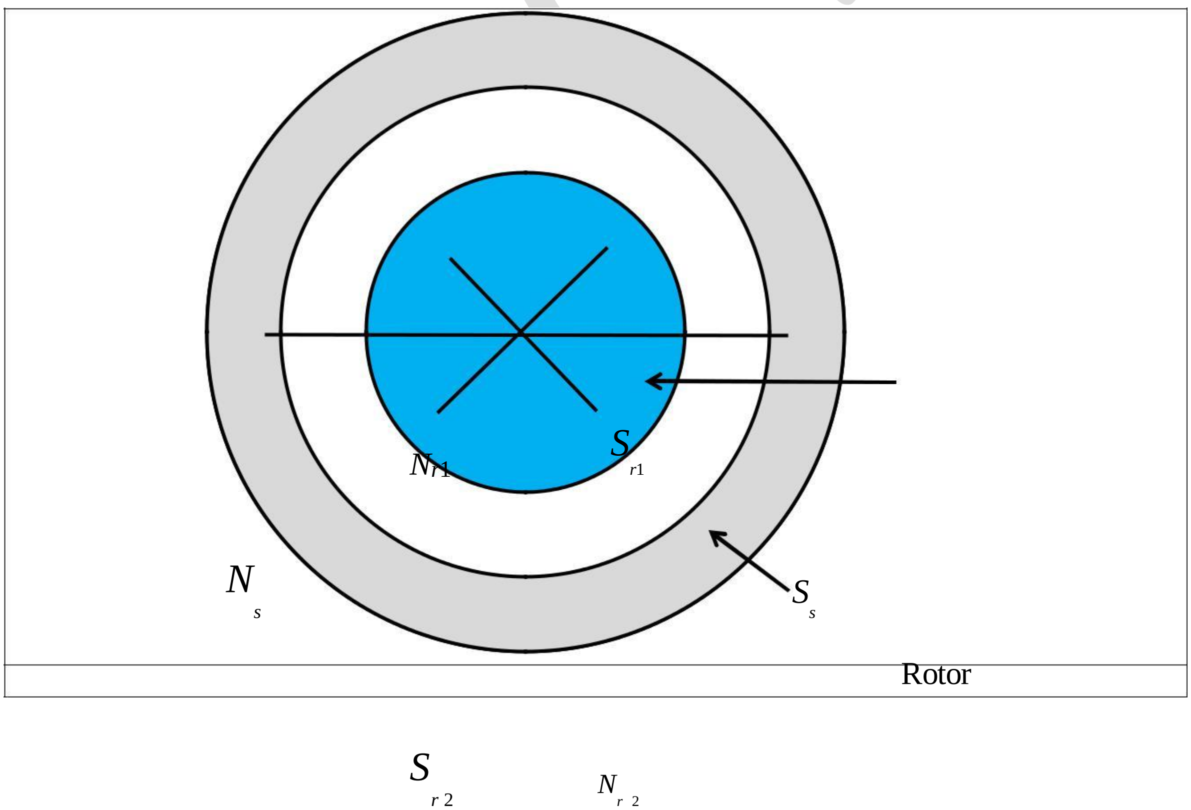
Figure 2b: Magnetic field when rotor axis is not aligned with stator poles

To realign the rotor from the position shown in **Figure 2a** to position shown in **Figure 2b**, a torque is exerted by the flux lines on the rotor. This torque is known as the ***reluctance*** or ***alignment torque***.

.

Why Should the Number of Poles on Stator Equal to the Number of Poles on Rotor?

In the previous section it has been shown that to produce electromagnetic torque, the magnetic field produced by the stator has to interact with the magnetic field produced by the rotor. However, ***if the number of poles producing the stator magnetic field is not equal to the number of rotor poles producing the rotor magnetic field, then the net torque produced by the motor will be zero***. This is illustrated by the motor configuration shown in **Figure 3**. In this motor the stator has two poles (N_s , S_s) and the rotor has four poles (N_{r1} , S_{r1} , N_{r2} , S_{r2}). The angle between the stator poles is 180^o and the angle between the rotor poles is 90^o. From the arrangement shown in **Figure 3** it can be seen that the angle between N_{r1} and N_s is equal to the angle between N_{r2} and S . Hence, a repulsive force exists between N_{r1} and N_s in clockwise direction and an attractive force exists between N_{r2} and S in the anticlockwise direction. Both, the attractive and repulsive forces are of same magnitude and the resultant of these forces is zero.



Stator

Figure 3: Configuration of a motor with unequal number of stator and rotor poles

Now consider the pole pairs (N_s , $S_{r\ 2}$) and (S_s , S_{r1}), the angle between the pole pairs is same. Hence, the force of attraction between N_s and $S_{r\ 2}$ is same as the force of repulsion between S_s and S_{r1} and thus, the resultant force acting on the rotor is zero. Therefore, in this case no electromagnetic torque is developed.

.

From the above discussion it can be seen that the resultant electromagnetic torque developed due to two stator poles and 4 rotor poles is zero. This leads to the conclusion that ***in all rotating electric machines, the number of rotor poles should be equal to number of stator poles for electromagnetic torque to be produced***.

How Continuous Torque is Produced by a Motor

In the previous section it has been seen that to produce electromagnetic torque, following two conditions have to be satisfied:

- δ Both stator and rotor must produce magnetic field
- δ The number of magnetic poles producing the stator magnetic field must be same as the number of magnetic poles producing the rotor magnetic field.

Now an important question that arises is ***how to create continuous magnetic torque***? To produce continuous torque the magnetic field of the stator should rotate continuously. As a result, the rotor's magnetic field will chase the stator's magnetic field and this result in production of continuous torque. This phenomenon is illustrated in **Figures 4a-4d**. In **Figure 4** a two pole machine is depicted and the rotors magnetic field is created by the permanent magnets. It is assumed that the stator's magnetic field rotates at a speed of 60 revolutions per minute (60 rpm) which is equivalent 1 revolution per second (1rps). To start the analysis it is assumed that at time $t = 0$, the stator's magnetic field axis aligns itself with the x – axis and the rotor's magnetic field axis makes an angle δ with the stator's magnetic axis (**Figure 4a**). At time $t = 0.25s$, the stator's magnetic field moves by 90^o and the rotor's magnetic field chases the stator's field and aligns as shown in **Figure 4b**. Similarly the locations of the magnetic field axis at time $t = 0.5s$ and $t = 0.75s$ are shown in **Figures 4c** and **Figure 4d**.

From the above discussion and observing **Figure 4** the following conclusions can be drawn:

- Σ The rotor's magnetic field chases the stator's magnetic field.

T The angle (δ) between the stator's magnetic axis and the rotor's magnetic axis remains constant. Hence, the rotor's speed of rotation is same as that of the stator's magnetic field.

However, an important question that still remains unanswered is ***How to create a rotating magnetic field?***

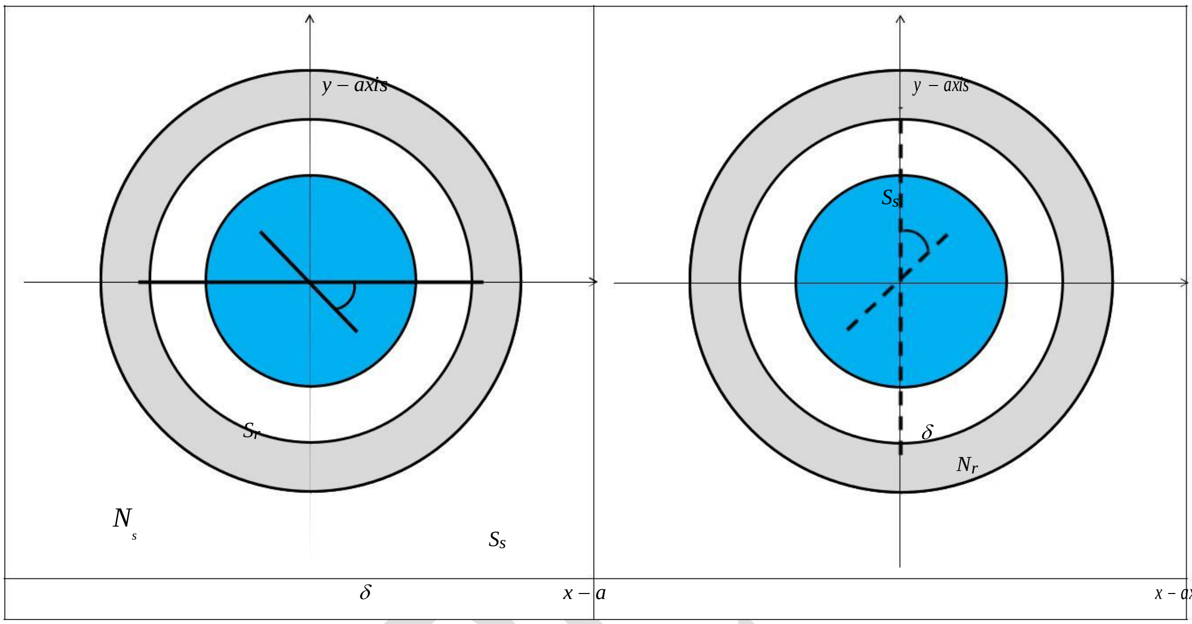


Figure 4a: Stator's magnetic field at time $t = 0$

Figure 4b: Stator's magnetic field at time $t = 0.25$

y − axis

y − axis

N_s

N_r

S_r

δ

S_s

N_s

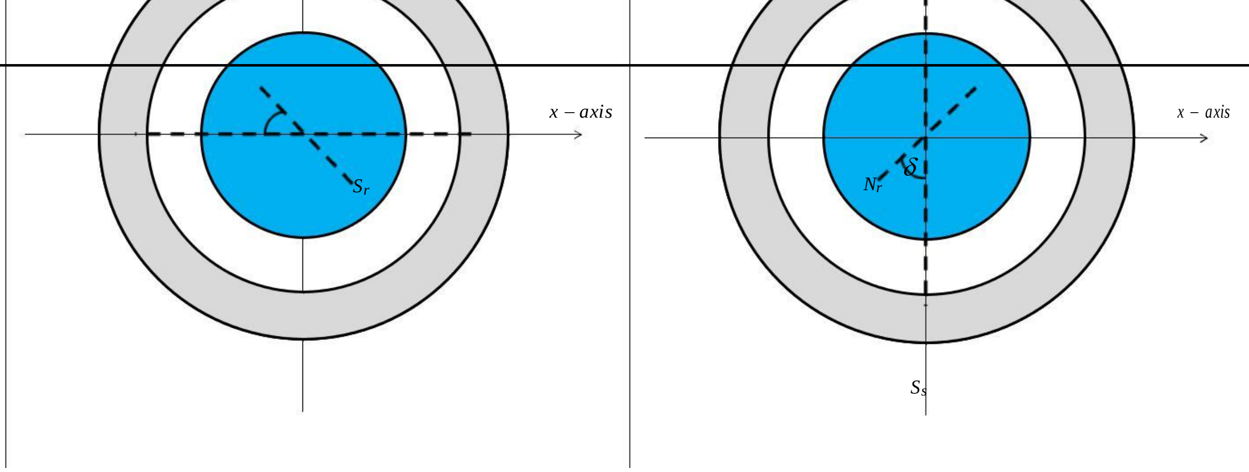


Figure 4c: Stator's magnetic field at time $t = 0.5$ **Figure 4d: Stator's magnetic field at time** $t = 0.75$

Rotating Magnetic Field

To understand the rotations of magnetic field consider a 2-pole 3-phase stator as shown in **Figure 5a**. The three phase windings ***a, b*** and ***c*** are represented by three coils aa', bb' and cc'. A current in phase ***a*** winding establishes magnetic flux directed along the magnetic axis of coil aa'. Similarly, the currents in phase ***b*** and ***c*** windings will create fluxes directed along the magnetic axes of coils bb' and cc' respectively. The three phase currents flowing the winding is shown in **Figure 5a**. At time instant **1**, the currents of each phase are

$$i_a = I_m \; ; \; i_b = -\frac{I_m}{2} \; ; \; i_c = -\frac{I_m}{2} \tag{2}$$

where

I_m = maximum value of the current

Since, i_b and i_c are negative, crosses must be shown in coil-sides b' and c' and dots in the coil sides b and c .The right hand thumb rule gives the flux distribution as shown in **Figure 5b**. In **Figure 5b** and the following figures, the thicker line indicates higher magnitude to flux. The

At instant **2**, the currents are

$$i_a = \frac{I_m}{2} \; ; \; i_b = \frac{I_m}{2} \; ; \; i_c = -I_m \tag{3}$$

The magnetic flux distribution created by the currents at instant **2** is shown in **Figure 5c**. Eventually at instant **3**, the currents are

$$i_a = -\frac{I_m}{2};\; i_b = I_m;\; i_c = -\frac{I_m}{2} \qquad (4)$$

The magnetic flux distribution created by the phase currents given by **equation 4** is shown in **Figure 5d**. From **Figure 5b** to **5c** it can be seen that the 2 poles produced by the resultant flux are seen to have turned 60^{o}. At other instants of time, i.e. as time elapses, the two poles rotate further. In this manner a rotating magnetic field is produced. The space angle traversed by a rotating flux is equal to the time angle traversed by currents.

After having discussed the production of rotating magnetic field, an important issue that still remains unresolved is: ***How to create the second magnetic field that will follow the rotating magnetic field created by the stator?*** This question is answered in following section.

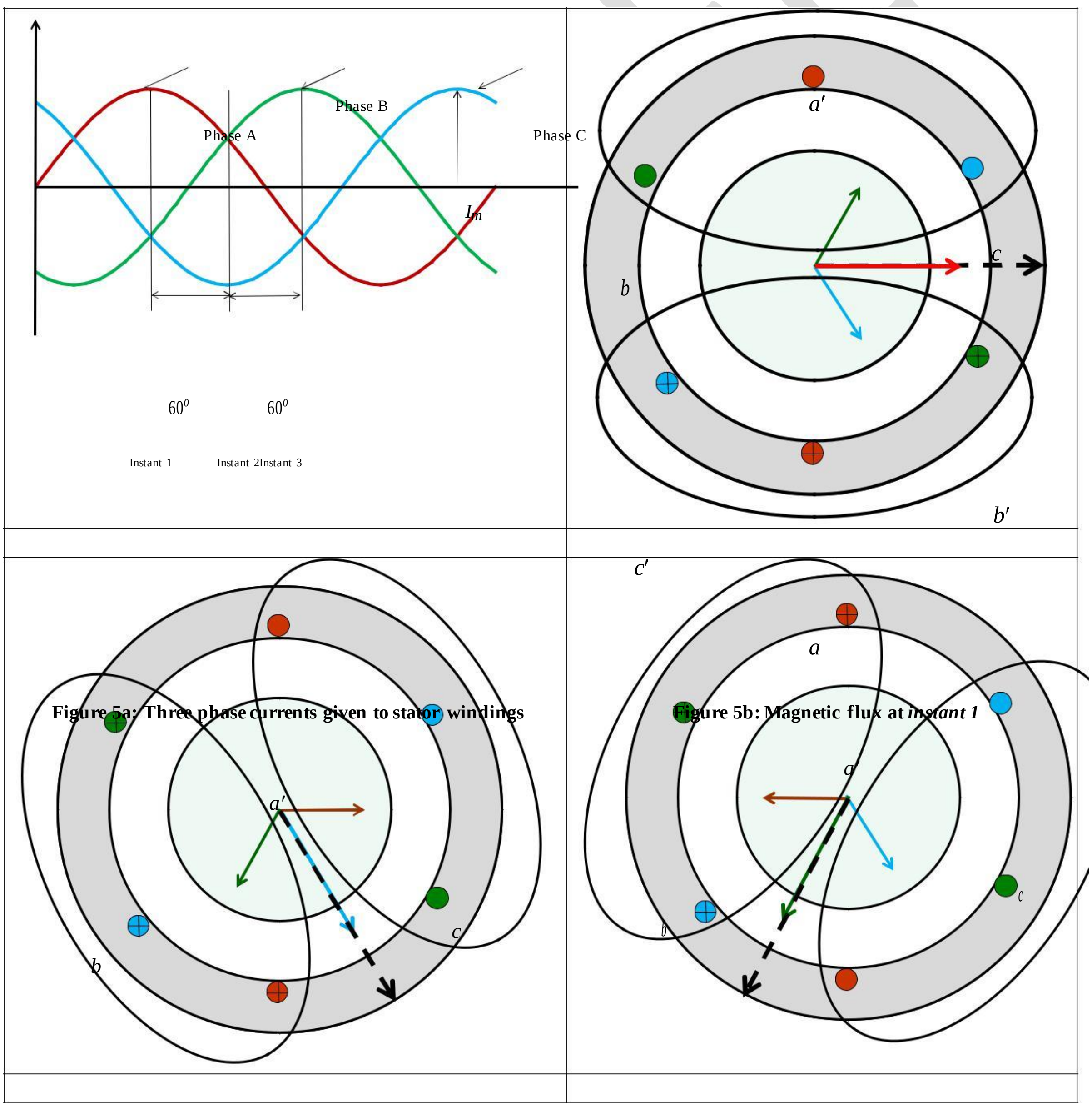


Figure 5a: Three phase currents given to stator windings

Figure 5b: Magnetic flux at *instant 1*

b'

c'

a

Figure 5c: Magnetic flux at *instant 2*

b'

c'

a

Figure 5d: Magnetic flux at *instant 3*

How to Create the Second Magnetic Field

From **equation 1** it can be seen that to produce torque two magnetic fields are required. The rotating magnetic field created by the stator has been discussed in the previous section and this section deals with the generation of rotor magnetic field. There are multiple ways to produce the rotor magnetic field namely:

- α Having windings on the rotor and exciting then with dc current to produce magnetic field (known as ***Synchronous Machines***).
- α Having permanent magnets on the rotor to produce the rotor magnetic field (known as ***Permanent Magnet Synchronous Machines***).
- α Utilize the Faradays law of induction to induce electromotive force (e.m.f) in the rotor coils. The induced e.m.f will result in flow of current through the rotor conductors and these currents will produce a magnetic field. These machines are known as ***Induction Machines*** or ***asynchronous machines***.

Synchronous Machines

The general configuration of synchronous machine is shown in **Figure 6**. It can be seen from **Figure 6** that the rotor has a coil (denoted by a dot and a cross) and through this coil a dc current flows. Due to this dc current a pair of magnetic poles is created. The stator windings also create two magnetic fields that rotate with time and hence, the rotor's magnetic poles chase the stator's magnetic field and in the process electromagnetic torque is produced. The speed of rotation of rotor depends on the speed with which the stator's field rotates and hence, these machines are known as ***synchronous machine***.

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Permanent Magnet Synchronous Machines (PMSM)

In case of PMSM, the rotor field is created by permanent magnets rather than dc current passing through a coil (**Figure 7**). The principle of operation of PMSM is same as that of synchronous machine.

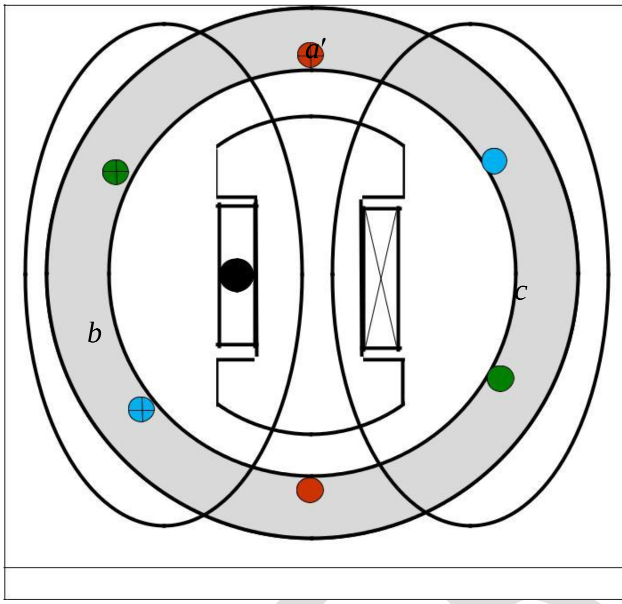

Figure 6: Synchronous machine

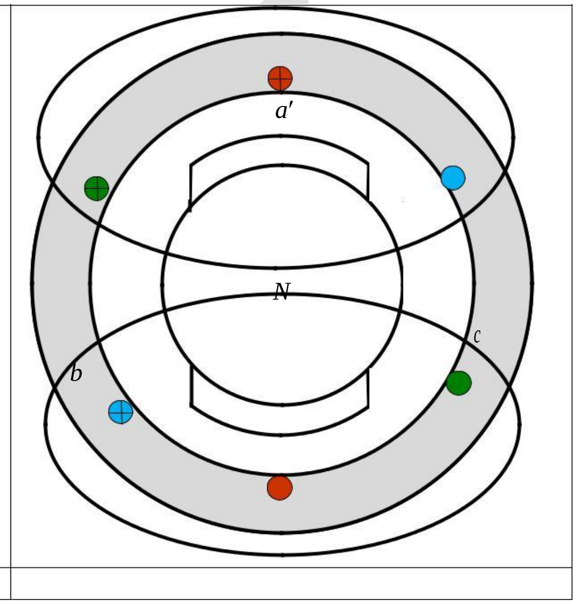

Figure 7: Permanent Magnet Synchronous machine

Induction Machine (IM)

Like synchronous machine, the stator winding of an induction machine is excited with alternating currents. In contrast to a synchronous machine in which a field winding on the rotor is excited with dc current, alternating currents flow in the rotor windings of an induction machine. In IM, alternating currents are applied to the stator windings and the rotor currents are produced by induction. The details of the working of the IMs are given in the following lectures.

After having discussed the general features of the electrical machines, the question that arises is***: how to analyse the machines?*** The analysis of electrical machines becomes simple by use of electrical equivalent circuits. The electrical equivalent circuits for the machines are discussed in the next section. One last concept that is relevant to electrical machines is principle of ***electrical*** and ***mechanical angle*** which is explained in the next section.

Electrical and Mechanical Angle

In **Figure 8**, it is assumed that the field winding is excited by a dc source and a coil rotates in the air gap at a uniform angular speed. When the conductor is aligned along $y - y'$ axis, the e.m.f induced is zero. Along $x - x'$ axis the induced e.m.f is maximum. In one revolution of the coil, the e.m.f induced is shown in **Figure 9**. If the same coil rotates in a 4 pole machine (**Figure 10**), excited by a dc source, the variation in the magnetic flux density and the induced e.m.f is shown in **Figure 11**. From **Figure 11** it can be seen that in one revolution of 360 mechanical degrees, 2 cycles of e.m.f (720 electrical degrees) are induced. The 720 electrical degrees in a 4 pole machine can be related to 360 mechanical degrees as follows

$$720 \text{ electrical degrees} = \frac{4}{2} \times (360 \text{ mechanical degrees}) \tag{5}$$

$$\Rightarrow \theta_{elec} = \frac{4}{2}\theta_{mech}$$

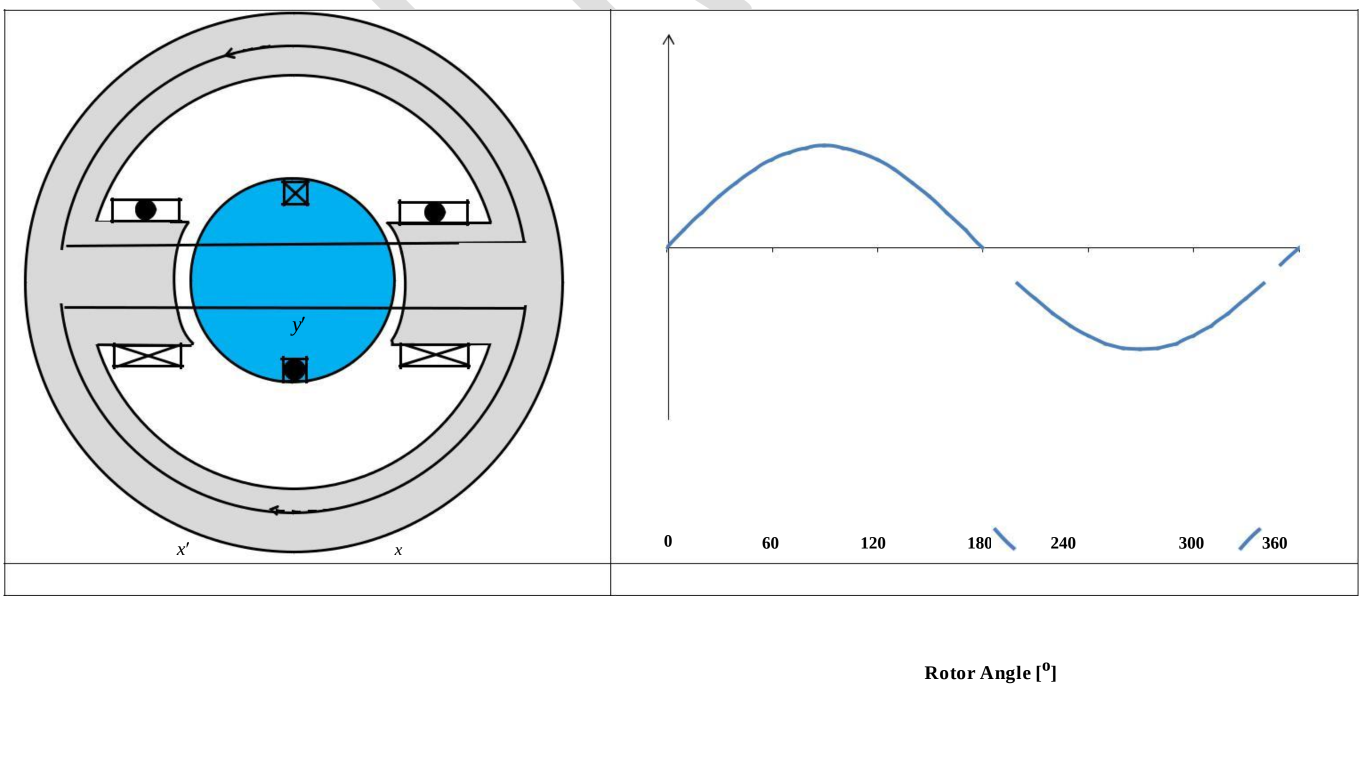

Figure 8: A two pole machine

Figure 9: Induced e.m.f in the rotor coils of a two pole machine

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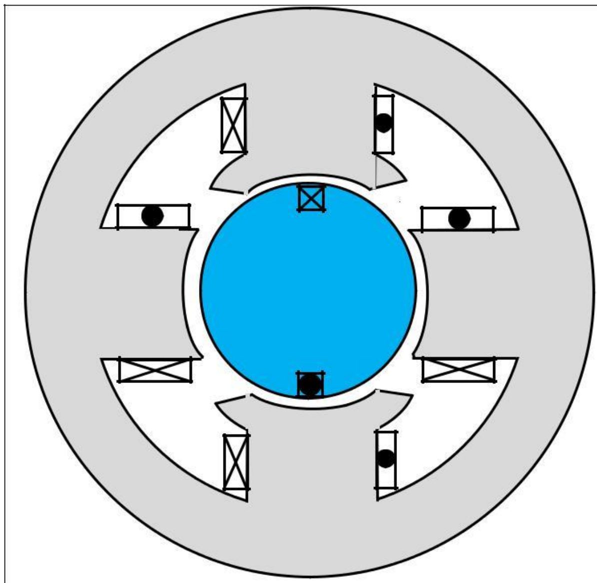


Figure 10: A four pole machine

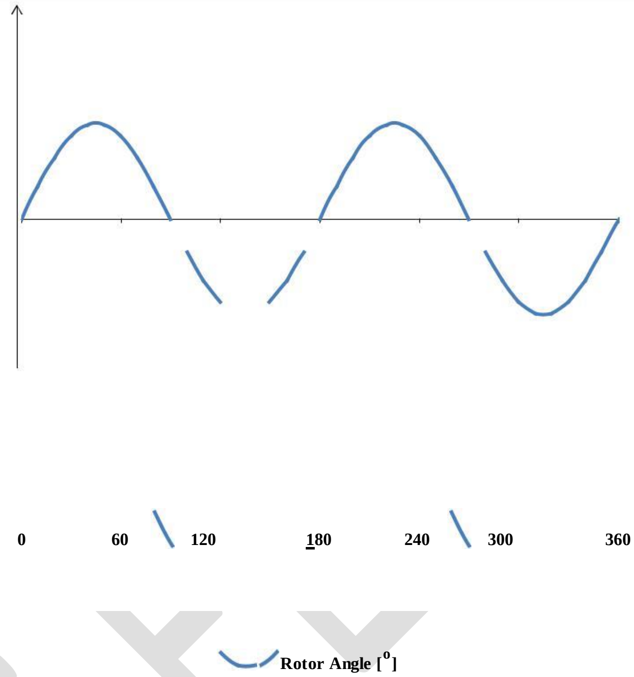


Figure11: Induced e.m.f in the rotor coils of a four pole machine

For a **P**-pole machine, **P/2** cycles of e.m.f will be generated in one revolution. Thus, for a **P** pole machine

$$\Pi._{elec} = \frac{P}{2}\theta_{mech}$$

$$\Rightarrow \frac{d\theta_{elec}}{dt} = \frac{P}{2}\frac{d\theta_{mech}}{dt} \qquad (6)$$

αα) $\omega_{elec} = \frac{P}{2}\,\omega_{mech}$

In a 4 pole, in one revolution 2 cycles of e.m.f are generated. Hence, for a **P** pole machine, in one revolution **P/2** cycles are generated. For a **P**-pole machine, in one revolution per second, **P/2** cycles per second of e.m.f will be generated. Hence, for a **P** pole machine, in **n** revolutions per second $\frac{P}{2} \times n$ cycles/second are generated. The quantity cycles/second is the frequency ***f*** of the generated e.m.f. Hence,

g) $= \frac{P}{2} \times n$ Hertz $\Rightarrow f = \frac{PN}{120}$ Hertz

where (7)

N = the speed in rpm

Suggested Reading:

3. M. G. Say, *The Performance and Design of Alternating Current Machines*, CBS Publishers, New Delhi

4. S. J. Chapman, *Electric Machinery Fundamentals*, McGraw Hill, 2005

Lecture 18: Induction motor drives, their control and applications in EV/HEVs

Induction Motor for EV and HEV Application

Introduction

The topics covered in this chapter are as follows:

- Traction Motors
- Principle of Operation of Induction Motor (Mathematical Treatment)
- Principle of Operation of Induction Motor (Graphical Treatment)
- Fluxes and MMF in Induction Motor
- Rotor Action
- Rotor e.m.f and Equivalent Circuit
- Complete Equivalent Circuit
- Simplification of Equivalent Circuit
- Analysis of Equivalent Circuit
- Thevenin's Equivalent Circuit

Principles of Operation of Induction Motor (Mathematical Treatment)

In **Figure 1** a cross section of the stator of a three phase, two pole induction motor is shown. The stator consists of three blocks of iron spaced at 120° apart. The three coils are connected in Y and energized from a three phase system. When the stator windings are energized from a three phase system, the currents in the coils reach their maximum values at different instants. Since the three currents are displaced from each other by 120° electrical, their respective flux contributions will also be displaced by 120° electrical. Let a balanced three phase current be applied to the stator with the phase sequence ***A-B-C***

$$I_A = I_m \cos\omega t$$

$$I_B = I_m \cos\left(\omega t - \frac{2\pi}{3}\right) \tag{1}$$

$$I_C = I_m \cos\left(\omega t - \frac{4\pi}{3}\right)$$

The instantaneous flux produced by the stator will hence be

$$\phi_A = \phi_m \cos\omega t$$

$$\phi_B = \phi_m \cos\left(\omega t - \frac{2\pi}{3}\right) \tag{2}$$

$$\phi_C = \phi_m \cos\left(\omega t - \frac{4\pi}{3}\right)$$

The resultant flux at an angle **θ** from the axis of phase ***A*** is

$$\phi_T = \phi_A \cos(\theta) + \phi_B \cos(\theta - \frac{2\pi}{3}) + \phi_C \cos(\theta - \frac{4\pi}{3}) \qquad (3)$$

Substituting **equation 2** into **equation 3** gives

$$\phi_T = \phi_A \cos(\theta)\cos(\omega t) + \phi_B \cos(\theta - \frac{2\pi}{3})\cos(\omega t - \frac{2\pi}{3}) + \phi_C \cos(\theta - \frac{4\pi}{3})\cos(\omega t - \frac{4\pi}{3}) \qquad (4)$$

$$\Rightarrow \phi_T = \frac{3}{2}\phi_m \cos(\theta - \omega t)$$

From **equation** 4 it can be seen that the resultant flux has amplitude of 1.5 ϕ_m , is a sinusoidal function of angle θ and rotates in synchronism with the supply frequency. Hence, it is called a ***rotating field***.

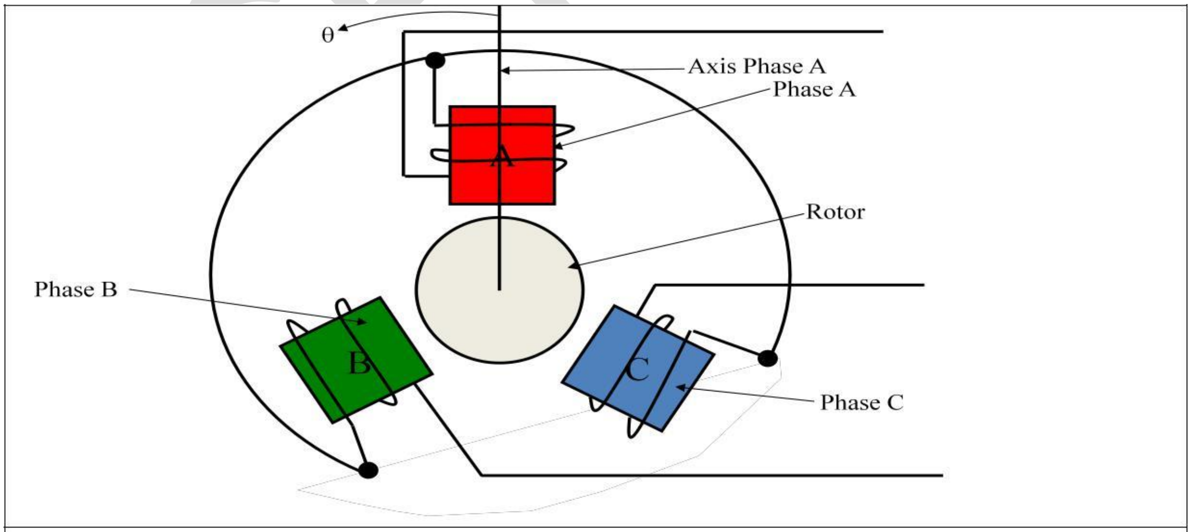


Figure 1: Cross section of a simple induction motor

Principles of Operation of Induction Motor (Graphical Treatment)

Let the synchronous frequency ω be 1rad/sec. Hence, the spatial distribution of resultant flux at t=0sec, t=60sec, t=120sec, t=180 sec, t=240sec and t=300sec and are shown in **Figure 2**. The explanation of the flux creation is as follows

- At t=0, phase **A** is a maximum north pole, while phase **B** and phase **C** are weak south poles, **Figure (2a)**.
- At t=60, phase **C** is a strong south pole, while phase **B** and phase **A** are weak north poles **Figure (2b)**.
- At t=120, phase **B** is a strong north pole, while phase **A** and phase **C** are weak south poles **Figure (2c)**.
- At t=180, phase **A** is a strong south pole, while phase **B** and phase **C** are weak north poles **Figure (2a)**.
- At t=240, phase **C** is a strong north pole, while phase **A** and phase **B** are weak south poles **Figure (2e)**.
- At t=300, phase **B** is s strong south pole, while phase **C** and phase **A** are weak north poles **Figure (2f)**.

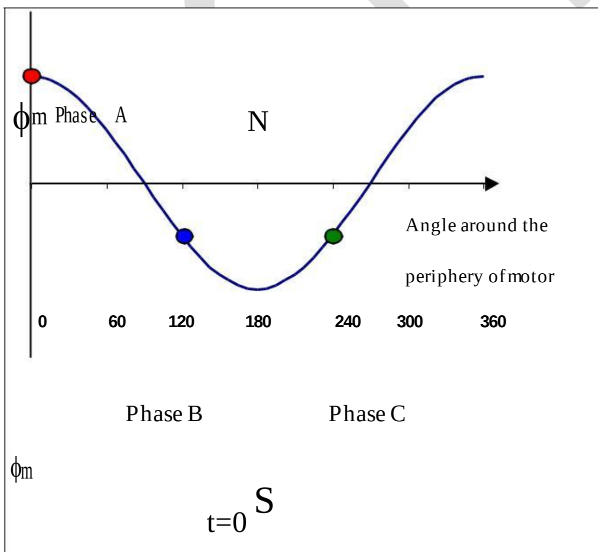


Figure 2a: Magnetic poles position at t=0

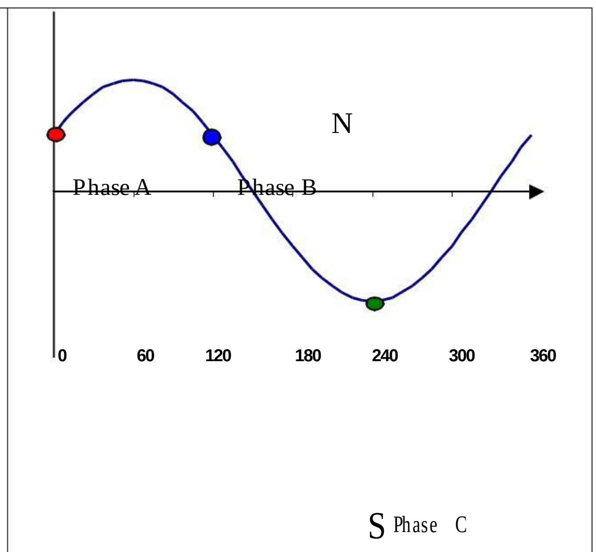
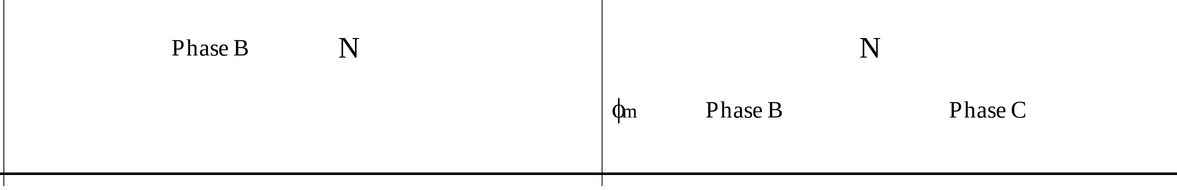


Figure 2b: Magnetic poles position at t=60



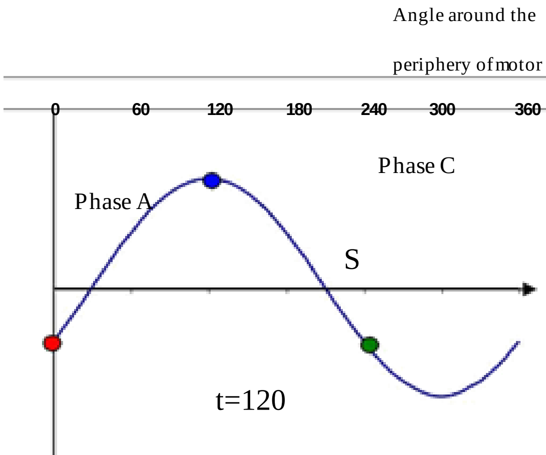


Figure 2c: Magnetic poles position at t=120

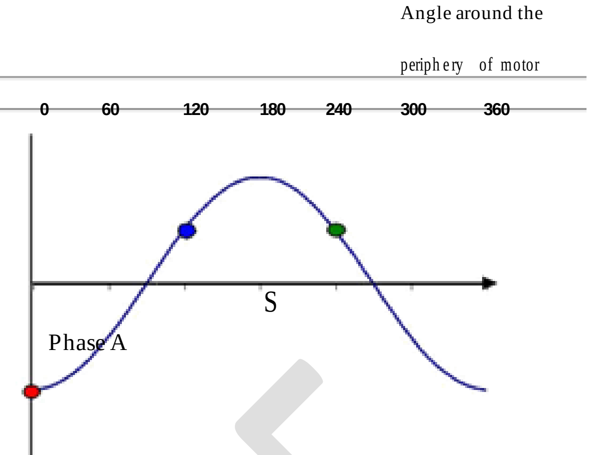


Figure 2d: Magnetic poles position at t=180

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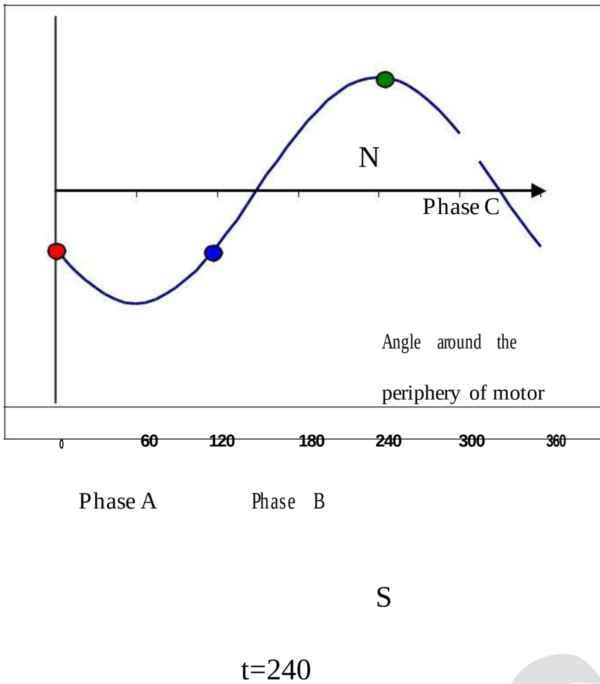


Figure 2e: Magnetic poles position at t=240

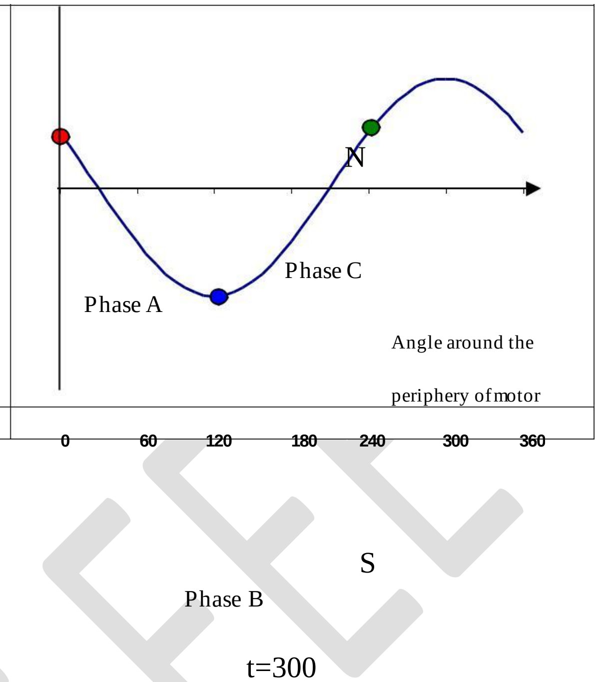


Figure 2f: Magnetic poles position at t=300

Fluxes and MMF in Induction Motor

Although the flux generated by each coil is only alternating flux, the combined flux contributions of the three coils, carrying current at appropriate sequential phase angles, produces a two pole rotating flux. The rotating flux produced by three phase currents in the stationary coils, may be linked to the rotating field produced by a magnet sweeping around the rotor (**Figure 3a**). The rotating field cuts the rotor bars in its anti clockwise sweep around the rotor. According to Lenz's law, the voltage, current and flux generated by the relative motion between a conductor and a magnetic field will be in a direction to oppose the relative motion. From **Figure 3a** it can be seen that the bars **a** and **b** are just under the pole centers and have maximum electromotive force (e.m.f) generated in them and this is indicated by large cross and dots. The bars away from the pole centers have reduced magnitude of generated e.m.fs and these are indicated by varying sizes of dots and crosses. If the rotor circuit is assumed purely resistive, then current in any bar is in phase with the e.m.f generated in that bar (**Figure 3a**). The existence of currents in the rotor circuit gives rise to rotor mmf F_2, which lags behind airgap flux ϕ_m by a space angle of 90°. The rotor mmf causes the appearance two poles N_2 and S_2 .The relative speed between the poles N_1 , S_1 and the rotor poles N_2 , S_2 is zero. Rotating pole N_1 repels N_2 but attracts S_2 . Consequently the electromagnetic torque developed by the interaction of the airgap flux ϕ_m and the rotor mmf F_2 is in the same direction as that of the rotating magnetic field (**Figure 3b**). The space phase angle between F_2 and ϕ_m is called the load angle and for this case it is 90° (**Figure 3b**). The torque produce is given by

$$T_e = k\phi F_2 \sin\left(\frac{\pi}{2}\right) = k\phi F_2 \tag{5}$$

.

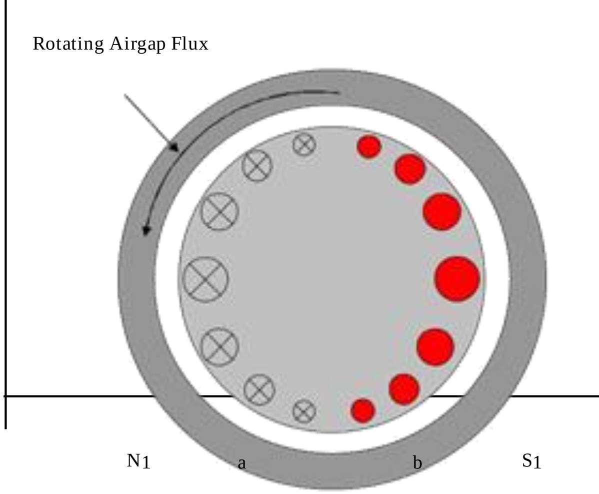


Figure 3a: Rotating airgap flux

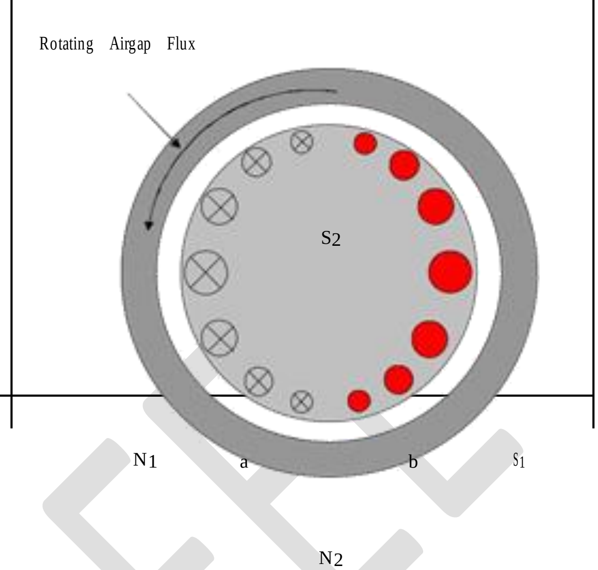


Figure 3b: Rotor and Stator fields

In actual machine, the rotor bars are embedded in the iron, hence the rotor circuit has leakage reactance. Thus the rotor current in each bar lags behind the generated e.m.f in that bar by rotor power factor angle:

$$\theta_2 = \tan^{-1}\frac{x_2}{r_2} \qquad (6)$$

From **Figure 4** it is seen that bars **a** and **b** under the poles have a maximum generated e.m.fs. On account of the rotor reactance (x_2), the currents in these bars will be maximum only when the poles N_1 , S_1 have traveled through an angle θ_2 (**Figure 4**). The rotor current

generates rotor mmf F_2 is space displaced from the air gap flux ϕ_m by a load angle $\theta_2 + \frac{\pi}{2}$. The torque produced by the motor in this situation is

$$T_e = k\phi F_2 \sin(\frac{\pi}{2} + \theta_2) \quad (7)$$

.

Greater the value of x_2 , greater is the departure of load angle from its optimal value of $\pi/2$

and lesser is the torque. To generate a high starting torque, θ_2 should be made as small as possible and this is done by increasing rotor resistance r_2 .

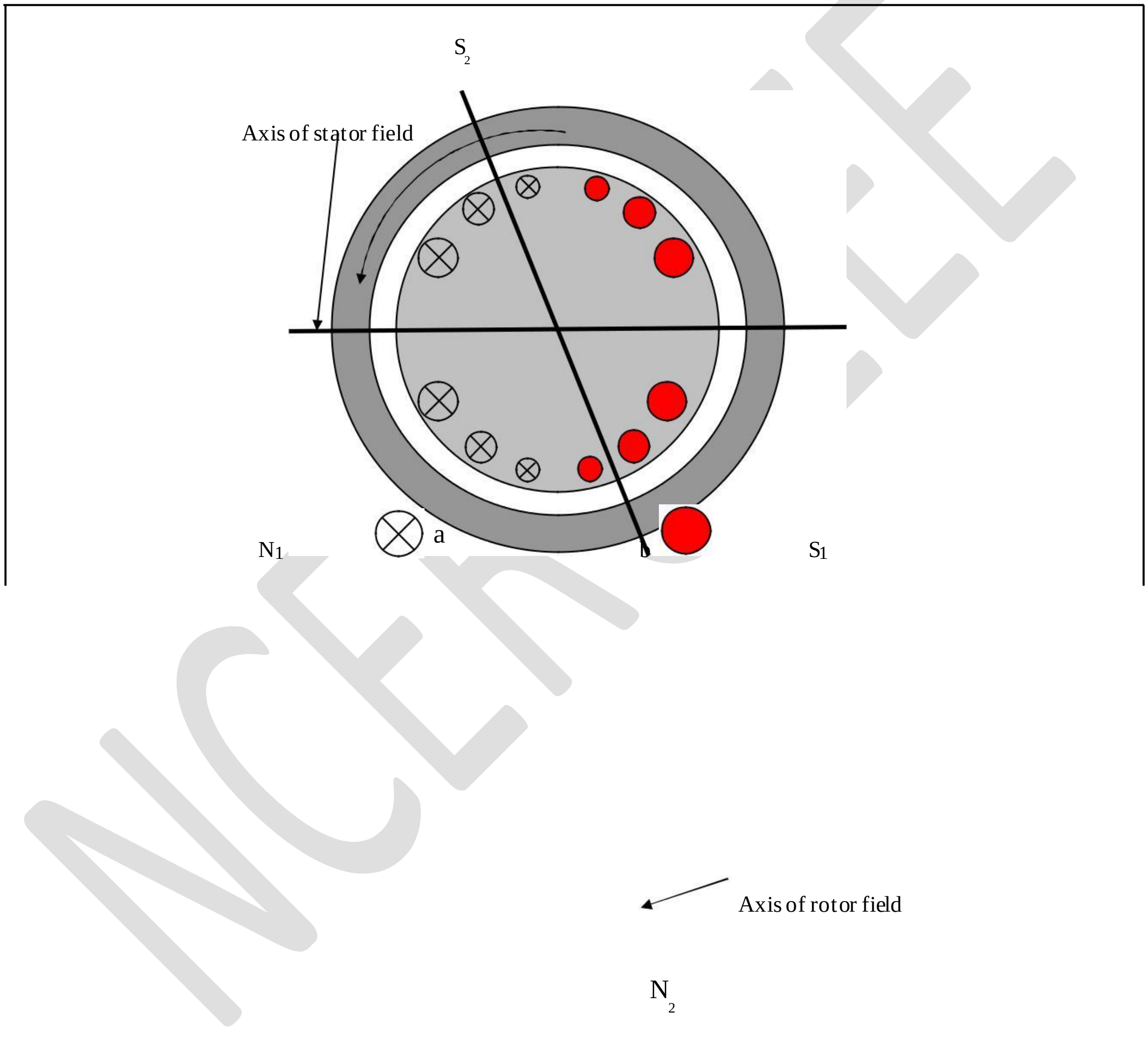


Figure 4: Axis of rotor and stator fields in case of rotor with inductance

Rotor Action

At standstill, rotor conductors are being cut by rotating flux wave at synchronous speed n_s . Hence, the frequency f_2 of the rotor e.m.f and current is equal to the input voltage frequency f_1 . When the rotor rotates at a speed of n_r rotations per second (r.p.s) in the direction of rotating flux wave, the relative speed between synchronously rotating flux and rotor conductors becomes ($n_s - n_r$) r.p.s, i.e.,

$$f_2 = \frac{P\,(n_s - n_r)}{2} \qquad (8)$$

where P is the number of poles of the machine

Hence, the slip of the machine is defined as

$$s = \frac{n_s - n_r}{n_s} \qquad (9)$$

Thus, the rotor frequency is defined as

$$f_2 = \frac{P \times s \times n_s}{2} = s f_1 \qquad (10)$$

At standstill the rotor frequency is f_1 and the field produced by rotor currents revolves at a speed equal to $2f_1/P$ w.r.t. rotor structure. When the rotor rotates at a speed of nr, the rotor frequency is sf_1 and the rotor produced field revolves at a speed of $2(sf_1)/P = sn_s$ w.r.t. rotor structure. The rotor is already rotating at a speed of n_r w.r.t. stator. Hence, the speed of rotor field w.r.t. to stator is equal to the sum of mechanical rotor speed nr and rotor field speed sn_s w.r.t. rotor. Hence, the speed of the rotor field with respect to stator is given by

$$n_r + sn_s = n_s(1 - s) + sn_s = n_s \text{ r.p.s} \quad (11)$$

The stator and rotor fields are stationary with respect to each other at all possible rotor speeds. Hence, a steady torque is produced by their interaction. The rotor of an induction motor can never attain synchronous speed. If does so then the rotor conductors will be stationary w.r.t. the synchronously rotating rotor conductors and hence, rotor m.m.f. would be zero.

Rotor e.m.f and Equivalent Circuit

Let the rotor e.m.f. at standstill be E_2. When the rotor speed is $0.4n_s$, the slip is 0.6 and the relative speed between rotating field and rotor conductors is $0.6n_s$. Hence, the induced e.m.f., per phase, in the rotor is

$$0.6\, n_s \frac{E_2}{n_s} = 0.6E_2 \quad (12)$$

In general, for any value of slip s, the per phase induced e.m.f in the rotor conductors is equal to sE_2. The other quantities of the rotor are given as

The rotor leakage reactance at standstill is $x_2 = 2\pi f_1 L_2$ (13a)

The rotor leakage reactance at any slip s is $2\pi s f_1 L_2 = s x_2 \Omega$ (13b)

The rotor leakage impedance at standstill is $\sqrt{r_2^2 + x_2^2}$ (13c)

At any slip s rotor leakage impedance is $\sqrt{r_2^2 + (s x_2)^2}$ (13d)

The per phase rotor current at standstill is $\dfrac{E_2}{\sqrt{r_2^2 + x_2^2}}$ (13e)

The per phase rotor current at any slip s is $\dfrac{sE_2}{\sqrt{r_2^2 + (s x_2)^2}} = \dfrac{E_2}{\sqrt{\left(\dfrac{r_2}{s}\right)^2 + (x_2)^2}}$ (13f)

.

Based on **equation 13f** the equivalent circuit of the rotor is shown in **Figure 5**.

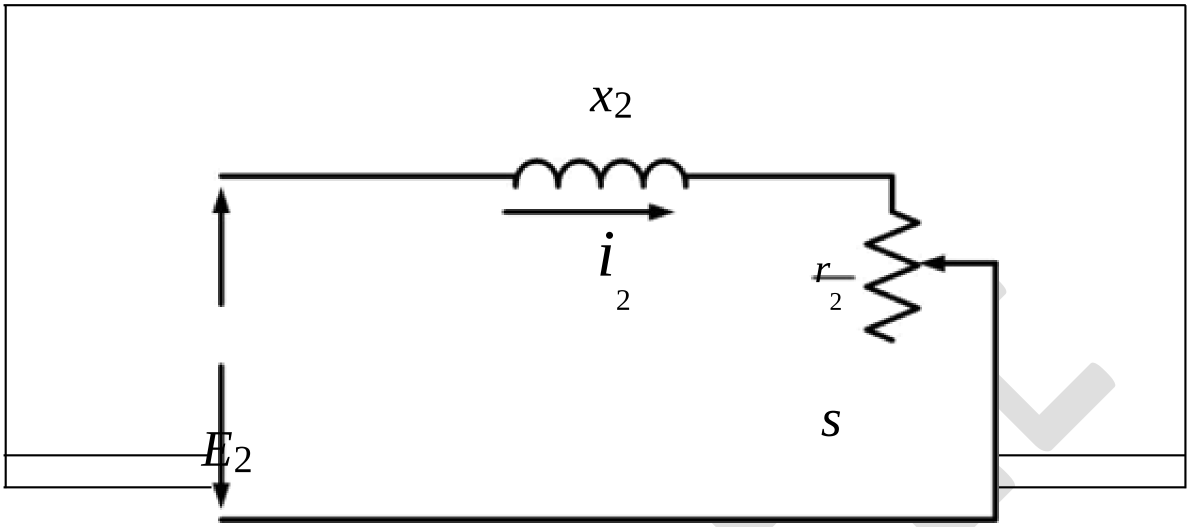


Figure 5: Equivalent circuit of rotor

Complete Equivalent Circuit

The rotating air gap flux generates back e.m.f. (E_1) in all the three phases of the stator. The stator applied terminal voltage V_1 has to overcome back e.m.f. E_1 and the stator leakage impedance drop:

$$V_1 = E_1 + I_1 (r_1 + jx_1) \quad (14)$$

The stator current I_1 consists of following two components, I_1' and I_m . The component I_1' is the load component and counteracts the rotor m.m.f. The other component I_m creates the resultant air gap flux ϕ_m and provides the core loss. This current can be resolved into two components: I_c in phase with E_1 and I_ϕ lagging E_1 by 90°. In the equivalent circuit of the stator shown in **Figure 6**, I_c and I_ϕ are taken into account by a parallel branch, consisting of core-loss resistance R_c in parallel to magnetizing reactance X_ϕ .

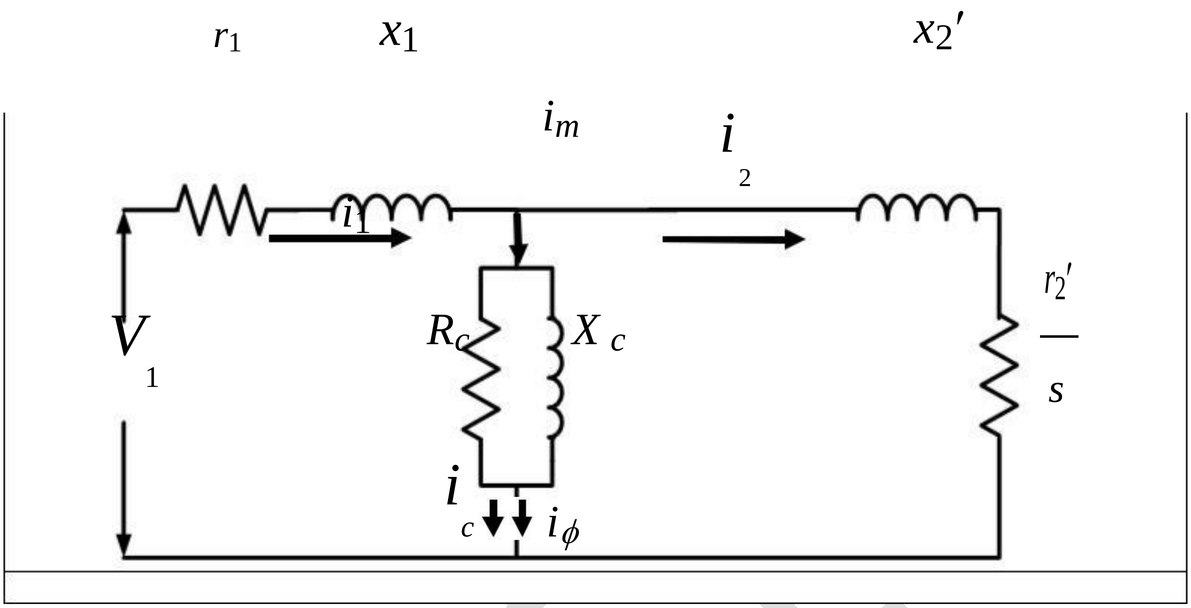


Figure 6: Equivalent circuit of stator

The rotor e.m.f. E_2 when referred to stator becomes

$$E_1 = \frac{N_1'}{N_2'} E_2 \qquad (15)$$

where N_1' and N_2' are number of turns in the stator and rotor respectively

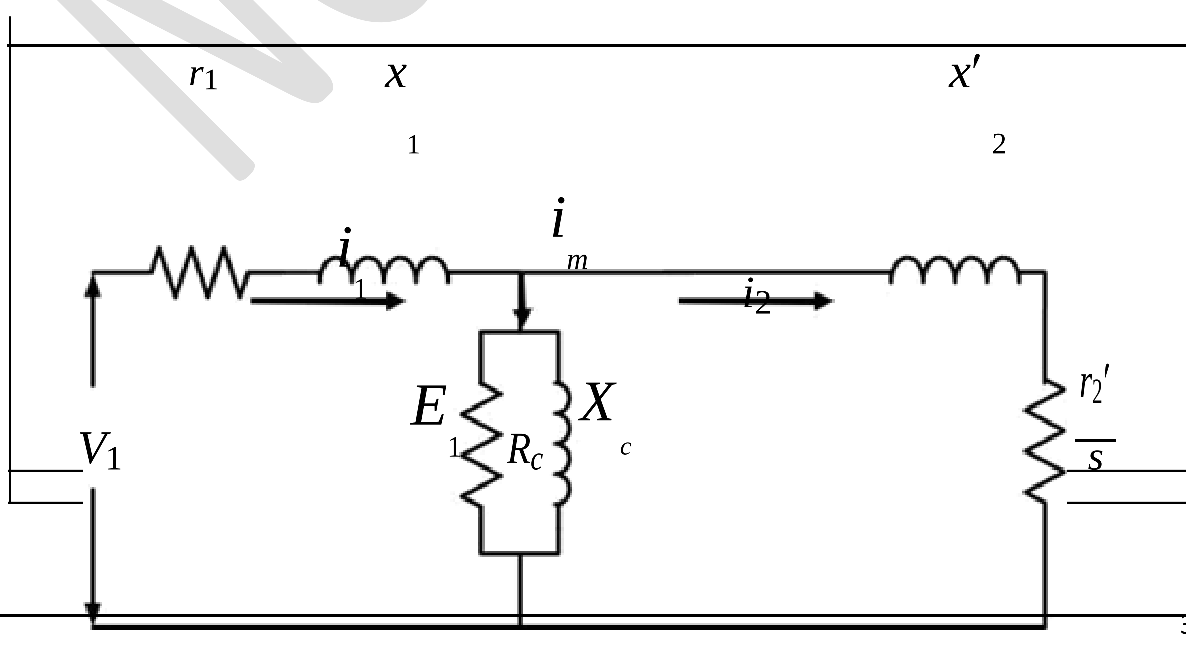
The rotor leakage impedance when referred to the stator is

$$Z_2' = \left(\frac{r_2}{s} + jx_2\right)\left(\frac{N_1'}{N_2'}\right)^2 = \frac{r_2'}{s} + jx_2' \qquad (16)$$

where

$$r_2' = \left(\frac{N_1'}{N_2'}\right)^2 r_2 ; \quad x_2' = \left(\frac{N_1'}{N_2'}\right)^2 x_2$$

After referred the rotor quantities towards stator, the combined equivalent circuit of the machine is shown in **Figure 7**. For simplicity the prime notations will not be used in the further discussions and all the rotor quantities henceforth will be referred to the stator side. Moreover, all the quantities are at stator frequency.



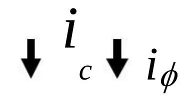


Figure 7: Complete equivalent circuit of Induction Motor

Simplification Equivalent Circuit

The use of exact equivalent circuit is laborious; hence some simplifications are done in the equivalent circuit. Under normal operating conditions of constant voltage and frequency, core loss in induction motors is usually constant. Hence, the core loss component can be omitted from the equivalent circuit, **Figure 8**. However, to determine the shaft power, the constant core loss must be taken into account along with friction, windage and stray load losses. It should be noted that all the quantities used in the equivalent circuit are per phase quantities. Steady state performance parameters of the induction motor, such as current, speed, torque, losses etc. can be computed from the equivalent circuit shown in **Figure 8**.

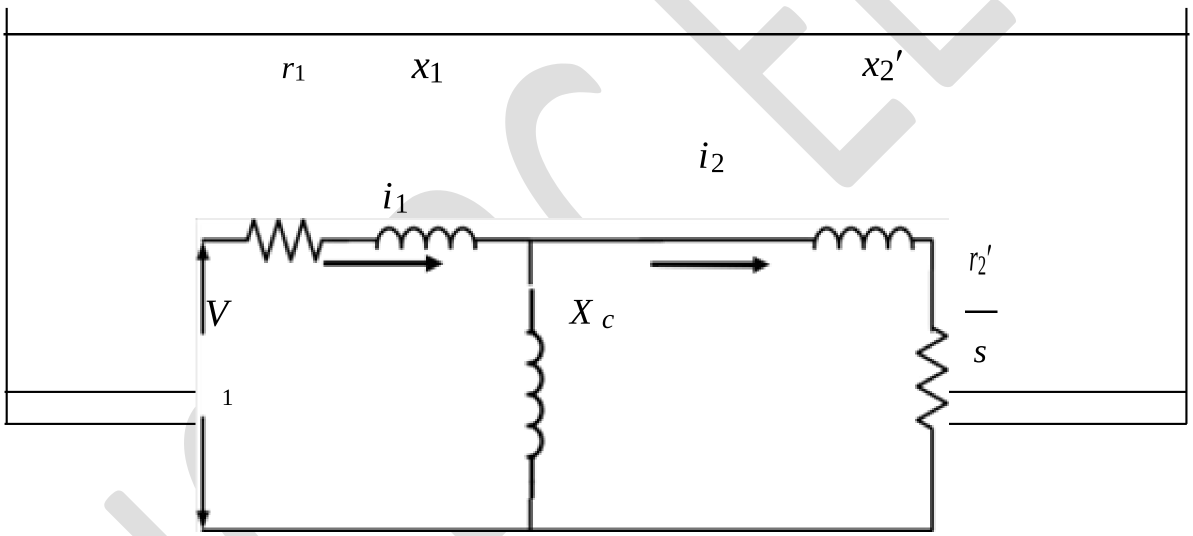


Figure 8: Simplified equivalent circuit of Induction Motor

Analysis of Equivalent Circuit

The total power transferred across the air gap (P_g) from the stator is

$$P_{gap} = n_{ph} i_2^2 \left(\frac{r_2}{s} \right)$$

$$P_{gap} = n_{ph}\left(i_2^2 r_2 + i_2^2 r_2\left(\frac{1-s}{s}\right)\right) \tag{17}$$

where n_{ph} is the number of phases

Hence, the rotor Ohmic loses and the internal mechanical power are given as

$$P_{rotor} = n_{ph} i_2^2 r_2 = n_{ph} sP_{gap} \tag{18}$$

$$P_{mech} = n_{ph} i_2^2 r_2 \left(\frac{1-s}{s}\right) \tag{19}$$

The internal (gross or air gap or the electromagnetic torque) torque developed per phase is given by

$$T_e = \frac{P_{mech}}{\omega_r} = \frac{(1-s)P_{gap}}{(1-s)\omega_s} = \frac{P_{gap}}{\omega_s} \tag{20}$$

where ω_r is the rotor speed and ω_s is the synchronous speed

The output or the shaft power is

$$P_{shaft} = P_{mech} - Mechanical\ losses$$

$$or \tag{21}$$

$$P_{shaft} = P_{gap} - Rotor\ Ohmic\ losses - Mechanical\ losses$$

Thevenin's Equivalent Circuit of Induction Motor

When the torque-slip or power-slip characteristics are required, application of Thevenin's theorem to the induction motor equivalent circuit reduces the computation complexity. For applying Thevenin's theorem to the equivalent circuit shown in **Figure 8**, two points ***a***, ***b*** are considered as shown in **Figure 9**. From these points the voltage source V_1 is viewed and the equivalent voltage at point ***a*** and ***b*** is

$$V_{eq} = \frac{V_1\left(jX_c\right)}{R_1 + j\left(X_1 + X_c\right)} \tag{22}$$

The equivalent impedance of the circuit as seen from points ***a*** and ***b*** is

$$Z_{eq} = \frac{\left(R_1 + jX_1\right)\left(jX_c\right)}{R_1 + j\left(X_1 + X_c\right)} \tag{23}$$

For most induction motors ($X_1 + X_c$) is much greater than R_1 . Hence, R_1 can be neglected from the denominator of **equation 22** and **equation 23**. The simplified expression for V_{eq} and Z_{eq} are

$$V_{eq} = \frac{V_1\left(jX_c\right)}{j\left(X_1 + X_c\right)} = \frac{V_1 X_c}{X_1 + X_c} \tag{24}$$

$$Z_{eq} = R_{eq} + jX_{eq} = \frac{R_1 X_c}{} + \frac{jX_1 X_c}{} \tag{25}$$

$eq \quad eq \quad eq$

$X_1 + X_c \quad X_1 + X_c$

From the Thevenin's equivalent circuit, the rotor current can be determined as

$$I_2 = \frac{V_{eq}}{\left(R_{eq} + \frac{r_2}{s}\right) + j\left(X_{eq} + X_2\right)} \qquad (26)$$

The airgap torque produced by the motor is

$$T_e = \frac{n_{ph}}{\omega_s}\frac{V_{eq}^2}{\left(R_{eq}+\frac{r_2}{s}\right)^2+\left(X_{eq}+X_2\right)^2}\frac{r_2}{s}$$

$$= \frac{K_t}{\left(R_{eq}+\frac{r_2}{s}\right)^2+X^2}\frac{r_2}{s} \qquad (27)$$

where

$$K_t = \frac{n_{ph}V_{eq}^2}{\omega_s} \text{ and } X = X_2 + X_{eq}$$

A typical torque versus slip curve for IM obtained from **equation 27** is shown in **Figure 10**.

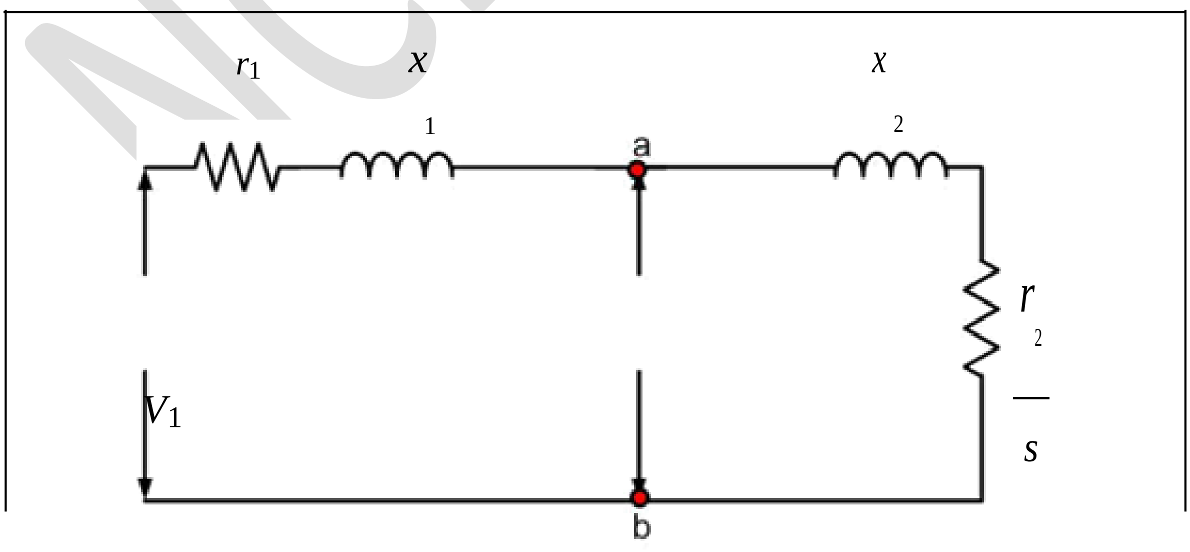


Figure 9: Thevenin's equivalent circuit

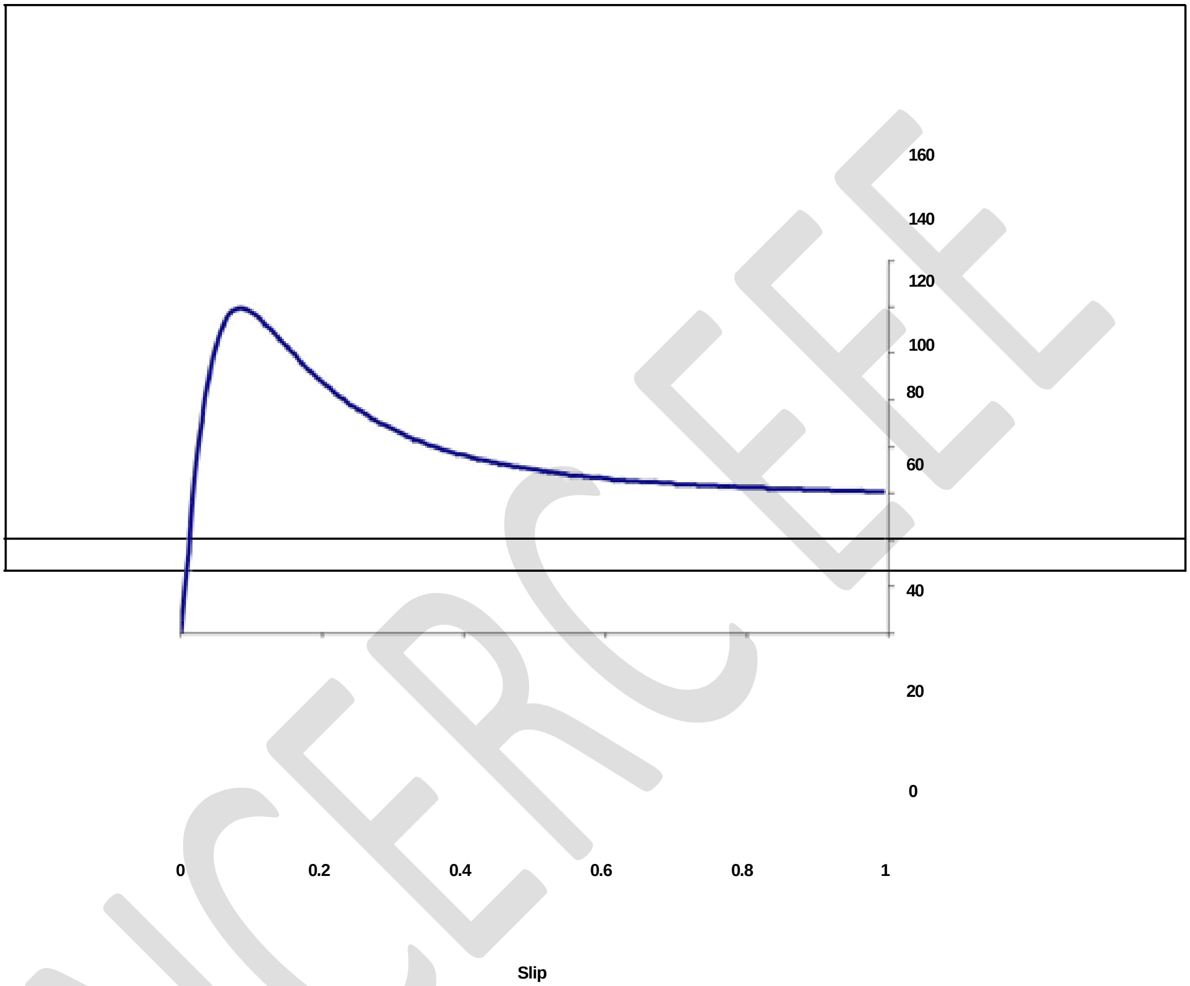


Figure 10: Torque vs. Slip curve of IM

Suggested Reading:

C M. G. Say, *The Performance and Design of Alternating Current Machines*, CBS Publishers, New Delhi

D S. J. Chapman, *Electric Machinery Fundamentals*, McGraw Hill, 2005

Lecture 19: Permanent magnet motors, their configurations and optimization

Control of Induction Motors

Introduction

The topics covered in this chapter are as follows:

- T Speed Control of Induction Motor
- Y Constant Volts/Hz control
- ς Implementation of Constant Volts/Hz Control
- Ω Steady State Analysis of IM with Constant Volts/Hz Control

Speed Control of Induction Motor (IM)

Speed control of IM is achieved in the inverter driven IM by means of variable frequency. Besides the frequency, the applied voltage needs to be varied to keep the air gap flux constant. The induced e.m.f in the stator winding of an ac machine is given by

$$E_1 = 4.44 k_{w1} \phi_m f_s N_1 \quad (1)$$

where

k_{w1} is the stator winding factor

ϕ_m is the peak airgap flux

f_s is the supply frequency

N_1 is the number of turns per phase in the stator

The stator applied terminal voltage V_1 (**Figure** 1) has to overcome back e.m.f. E_1 and the stator leakage impedance drop (refer Lecture 17):

$$V_1 = E_1 + i_1\,(r_1 + jx_1) \qquad (2)$$

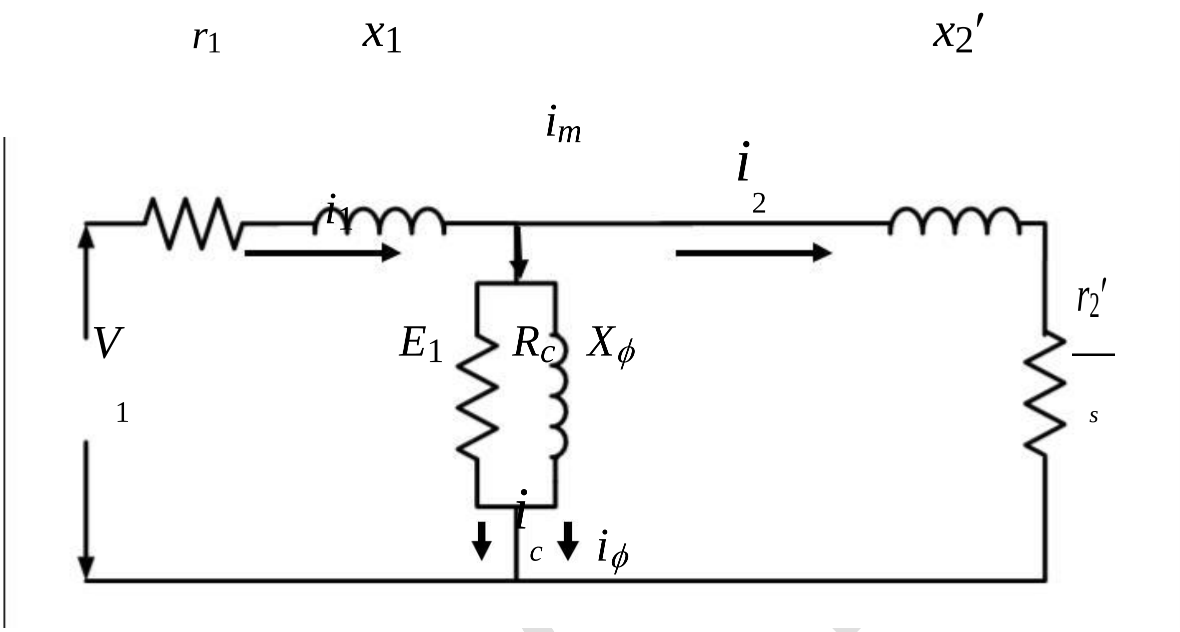


Figure 1: Equivalent circuit of IM

If the stator impedance ($r_1 + jx_1$) is neglected, the induced e.m.f approximately equals the supply phase voltage. Hence,

$$V_1 \cong E_1 \quad (3)$$

Substituting for E_1 from **equation 1** into **equation 2** gives the flux as

$$\phi_m \cong \frac{V_1}{K_b f_s} \quad (4)$$

where

$k_b = 4.44 k_{w1} N_1$ is the flux constant

Since the factor k_b is constant, from **equation 4** it can be seen that ***the flux is proportional to the ratio between the supply voltage and frequency***. Hence,

$$\phi_m \propto \frac{V_1}{f_s} \propto k_{vf} \quad (5)$$

where k_{vf} is the ration bewteen V_1 and f_s

From **equation 5**, it is seen that, to maintain the flux constant k_{vf} has to be maintained constant. Hence, whenever the stator frequency (f_s) is changed for speed control, the stator input voltage (V_1) has to be changed accordingly to maintain the airgap flux (ϕ_m) constant. A number of control strategies have been developed depending on how the voltage to frequency ratio is implemented:

- Constant volts/Hz control
- Constant slip-speed control
- Constant air gap control

- Vector Control

The constant volts/Hz strategy is explained in this lecture.

Constant Volts/Hz Control

The **equation 2** is converted into per unit (p.u) as

$$\frac{V_1}{V_b} = \frac{E_1 + I_1 (r_1 + jx_1)}{V_b}$$

$= V_{1n} = E_{1n} + I_{1n}(r_{1n} + jx_{1n})$ where

$$V_{1n} = \frac{V_1}{V_b};\quad I_{1n} = \frac{I_1}{I_b};\quad r_{1n} = \frac{I_b r_1}{V_b}$$

$$E_{1n} = \frac{E}{V_b} = \frac{jX_\phi i_\phi}{V_b} = \frac{j\omega_s L_\phi i_\phi}{V_b} = \frac{j\omega_s L_\phi i_\phi}{\lambda_b \omega_b} = j\left(\frac{\lambda_\phi}{\lambda_b}\right)\left(\frac{\omega_s}{\omega_b}\right)$$

$$x_{1n} = \frac{I_b x_1}{V_b} = \frac{I_b \omega_s L_{1s}}{V_b} = L_{1n}\omega_{sn}$$

$V_b = \lambda_b \omega_b$

V_b is the base voltage

I_b is the base current

λ_b is the flux linkage (flux linkage is rate of change of flux with repect to time) Hence, **equation 2** in p.u form is written as

$$V_{1n} = E_{1n} + i_{1n}(r_{1n} + jx_{1n}) = i_{1n} r_{1n} + j\omega_{sn}\left(L_{1n} i_{1n} + \lambda_{\phi n}\right)$$

where

$E_{1n} = jL_{\phi n}\omega_{sn} i_{\phi n} =$

$\lambda_{\phi n} = jL_{\phi n} i_{\phi n}$

The magnitude of the input voltage (V_1) is given as

$V_{1n} = \sqrt{(i_{1n} r_{1n})^2 + \omega_{sn}^2 (L_{1n}}$

$$\sqrt{\left(|i_{1n}| + \lambda\phi_n\right)^2} \qquad (6)$$

(8)

(7)

For a constant air gap flux linkages of 1pu, the pu applied voltage vs. p.u stator frequency is shown in **Figure 2**. The values of r_{1n} and x_{1n} used to obtain the plot of **Figure 2** are 0.03 and 0.05 pu respectively.

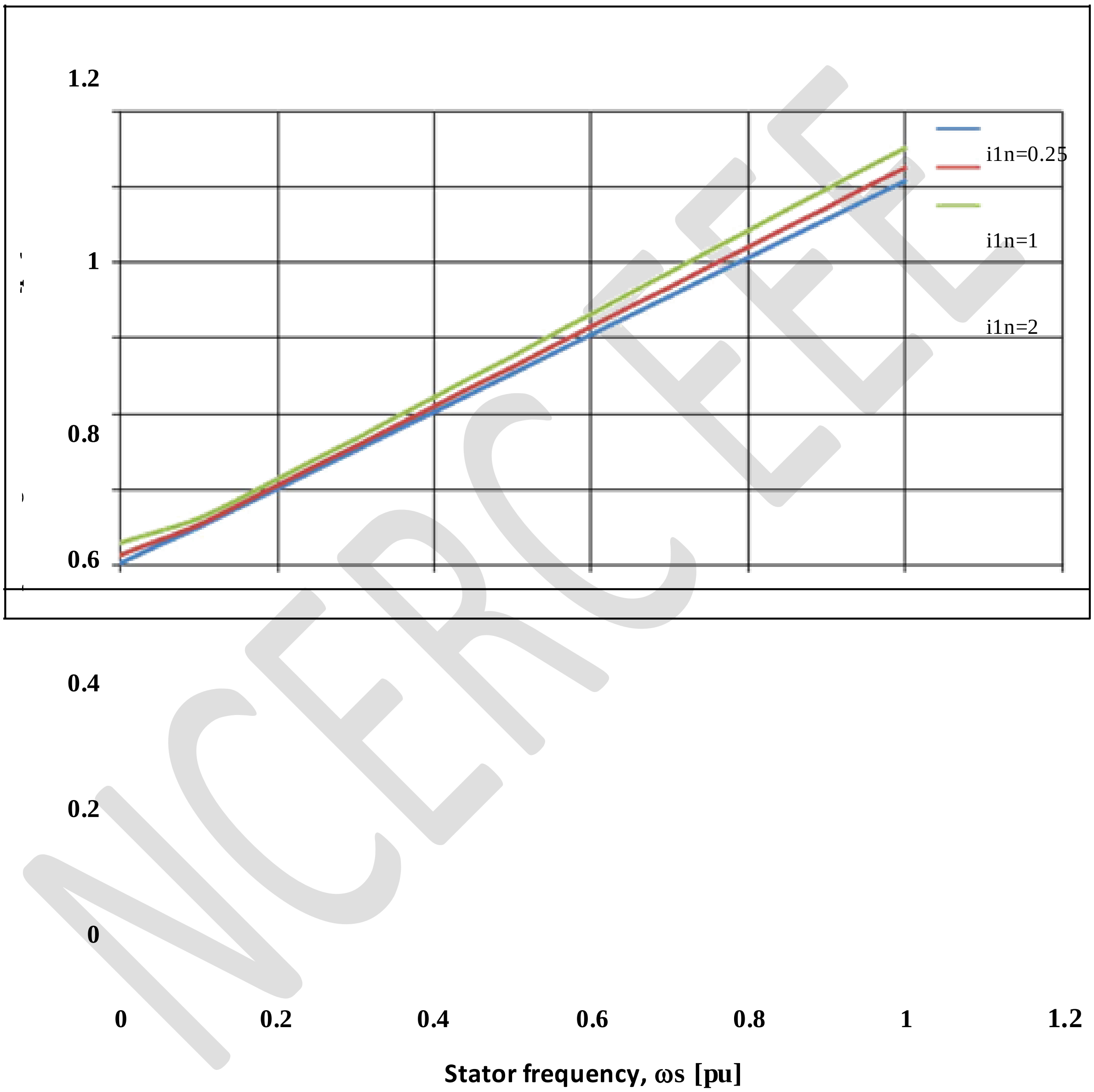


Figure 2: PU stator phase voltage vs. pu stator frequency

From **equation 8** it can be seen that the volts/Hz ratio needs to be adjusted in dependence on the frequency, the air gap flux magnitude, the stator impedance and the magnitude of the stator current. The relationship between the applied phase voltage and the frequency is written as

$$V_{1n} = V_{on} + k_{vf} f_{sn} \qquad (9)$$

From **equation 8** the parameters V_o and k_{vf} is obtained as

$$V_{on} = i_{1n} r_{1n}$$

(10)

$$k_{vf} = \omega_{sn} \left(\lambda_{\phi n} + L_{1n} i_{1n} \right)$$

The parameter V_o is the offset voltage required to overcome the stator resistive drop. In case the IM is fed by a DC-AC converter, the fundamental r.m.s phase voltage for 180^o conduction is given by (refer **Lecture 15**):

$$V_1 = \frac{v_{as}}{\sqrt{2}} = \frac{2}{\pi}\frac{V_{dc}}{\sqrt{2}} = 0.45V_{dc}$$

where (11)

V_{dc} is the input dc voltage to DC-AC converter

The **equation 11** can be written in pu form as

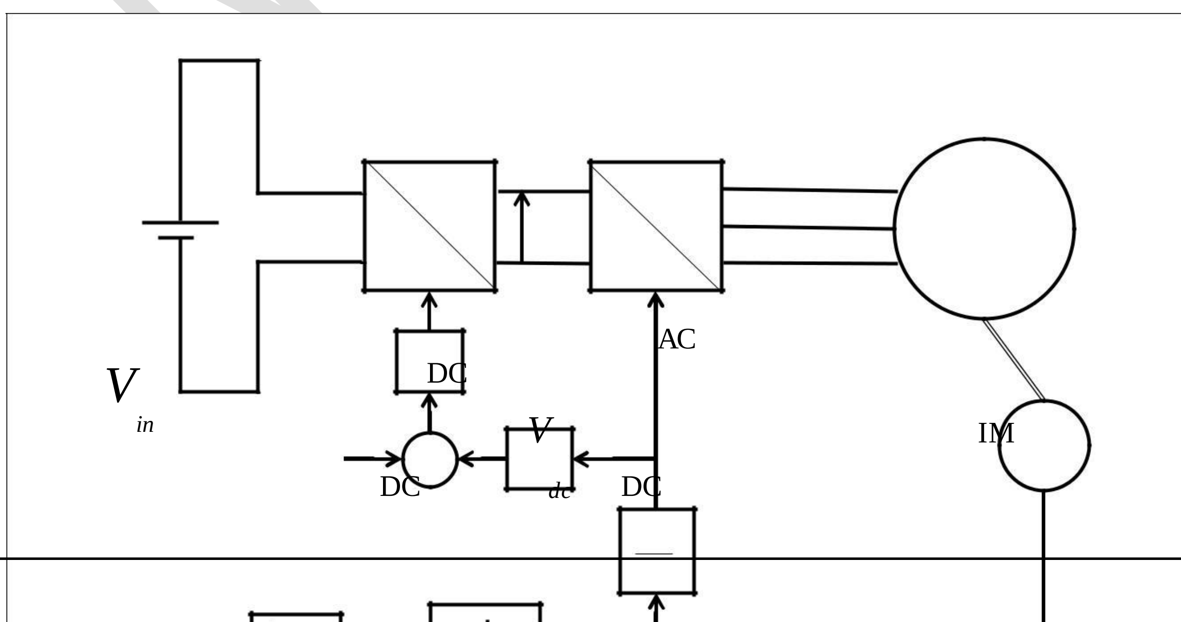
$$V_{1n} = \frac{V_1}{V_b} = 0.45V_{dcn} \qquad (12)$$

Substituting the value of V_{1n} into **equation 9** gives

$$0.45V_{dcn} = V_{on} + k_{vf}\, f_{sn}$$

Implementation of Constant Volts/Hz Control

The implementation of volts/Hz strategy is shown in **Figure 3**.



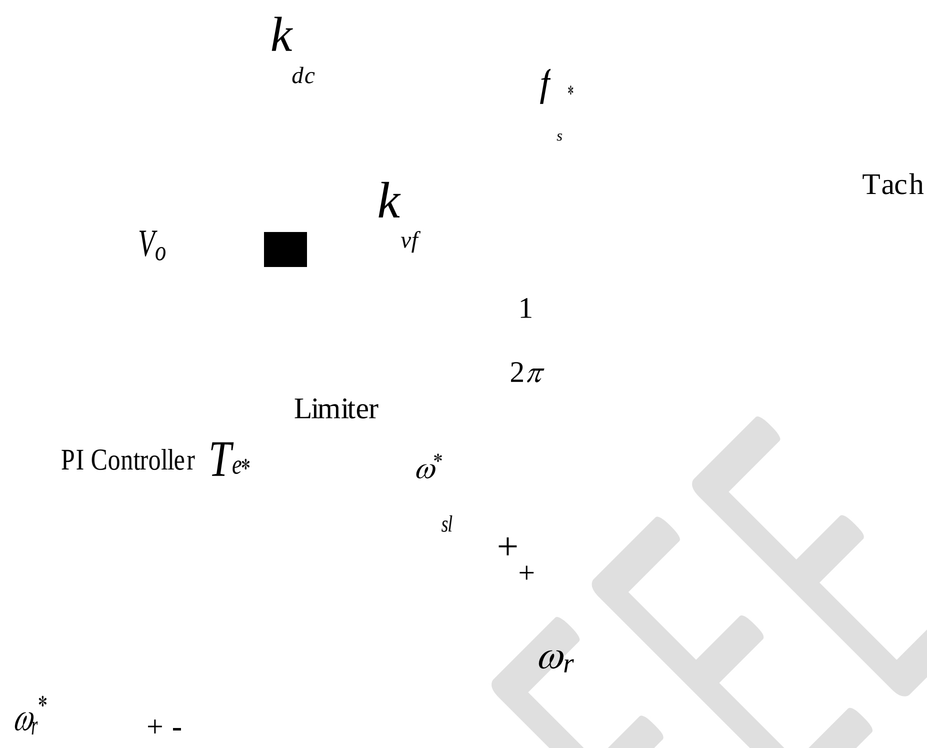


Figure 3: Closed loop induction motor drive with constant volts/Hz control strategy [1]

The working of the closed loop control shown in **Figure 3** is as follows:

- The actual rotor speed (ω_r) is compared with its desired value ω_r^* and the error is passed through a PI controller.
- The output of the PI controller is processed through a limiter to obtain the slip-speed command ω_{sl}^* . The limiter ensures that ω_{sl}^* is within the maximum allowable slip speed of the induction motor.

[6] The slip speed command is added to electrical rotor speed ω_r to obtain the stator frequency command f_s^* .

[7] The frequency command f_s^* is enforced in the inverter and the corresponding dc link voltage (V_{dc}) is controlled through the DC-DC converter.

[8] The offset voltage V_1^* is added to the voltage proportional to the frequency and multiplied by k_{dc} to obtain the dc link voltage.

Steady State Performance of IM with Constant Volts/Hz Control

The steady state performance of the constant-volts/Hz controlled induction motor is computed by using the applied voltage given in **equation 9**. Using the equivalent circuit of IM, the following steps are taken to compute the steady state performance:

[11] Start with a minimum stator frequency and a very small slip

[12] Compute the magnetization, core-loss, rotor and stator phase current

[13] Calculate the electromagnetic torque, power, copper and core losses

[14] Calculate the input power factor and efficiency.

[15] Increase the slip and go to ***step b*** unless maximum desired slip is reached.

[16] Increase the stator frequency and go to ***step a*** unless maximum desired frequency is reached.

In **Figure 4** the characteristics of volts/Hz control of an IM is shown. The parameters of the IM are as follows:

Applied stator line to line voltage $V_{ll} = 200V$

Frequency of applied voltage $f_s = 50Hz$

Rated Output Power $P_{out} = 3kW$

Stator resistance $r_1 = 0.3\Omega$

Stator leakage inductance $L_1 = 0.001H$

Rotor resistance $r_2 = 0.2\Omega$

Rotor leakage inductance $L_2 = 0.0015H$

Efficiency $\eta = 0.8$

Power factor $pf = 0.85$

Connection of phases: Y

Based on the above parameters of the motor, the base quantities are determined as follows:

Base speed $\omega_{base} = 2\pi f_S = 2\times \pi \times 50 = 314.16\ rad/s$

Base voltage $V_{base} = V_{ph} = \frac{200}{\sqrt{3}} = 115.47\ V$

Base power P_{base} = Rated Output Power = 3000 W

Base current $I_{base} = \frac{P_{base}}{3\times V_{base}\times \eta \times pf \times V_{base}} = 12.74\ A$

Base Torque $T_{base} = \frac{P_{base}}{\omega_{base}} = \frac{3000}{314.16} = 9.55\ Nm$

After having calculated the base values, the torque produced by the IM is calculated using the following expression (**equation 27** of Lecture 17):

$$T_e = \frac{n_{ph}}{\omega_s}\frac{V_{eq}^2}{\left(R_{eq}+\frac{r_2}{s}\right)^2+\left(X_{eq}+X_2\right)^2}\frac{r_2}{s} \quad (13)$$

The pu torque T_{en} is given by

$$T_{en} = \frac{T_e}{T_{base}} \quad (14)$$

In order to obtain the curve shown in **Figure 4**, the torque is calculated for different values of slip and frequency as described algorithm above. Using the constant volts/Hz control, the IM can be operated up to rated frequency. However, if it is required to operate the motor beyond rated speed then ***Flux weakening operation*** is used.

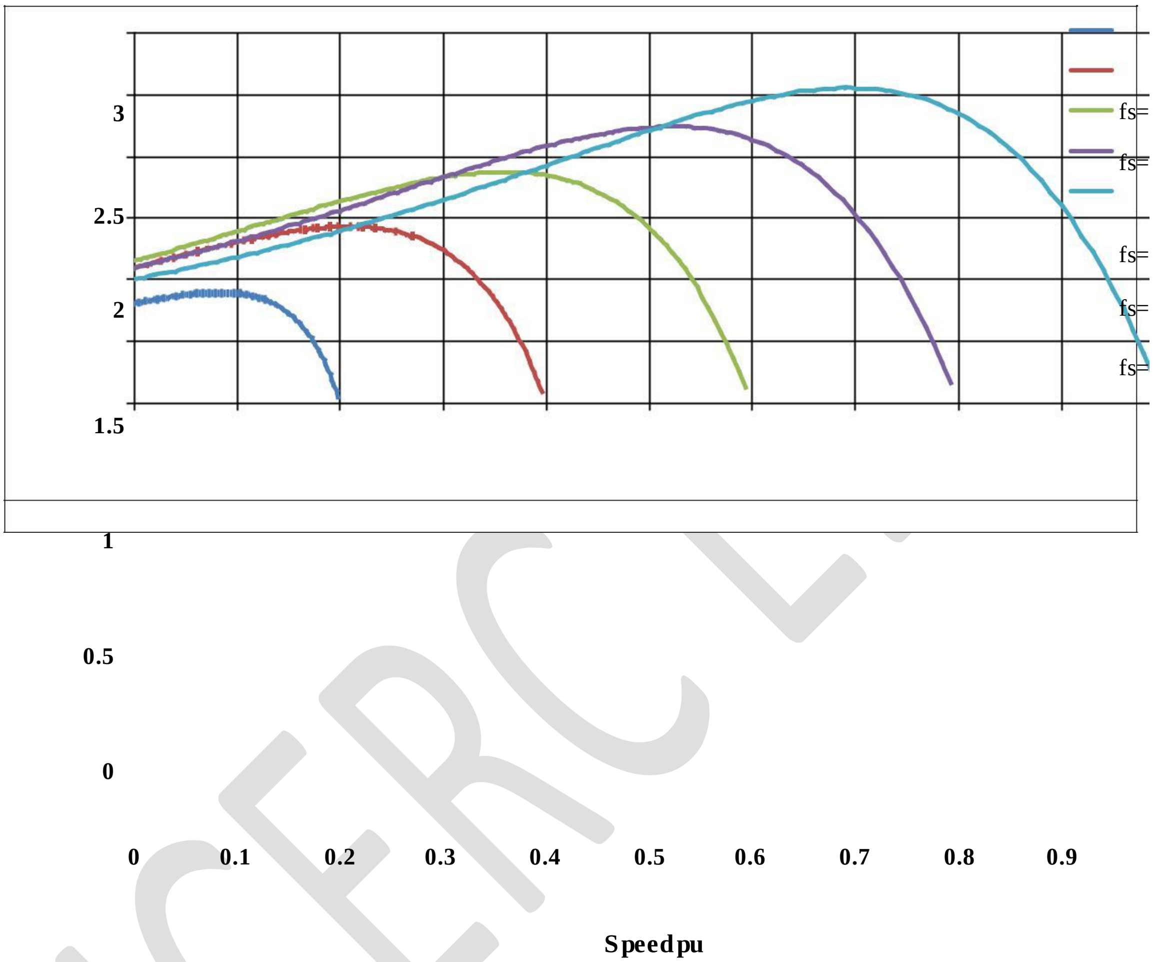


Figure 4: Torque vs. Speed Curve for Constant volts/Hz control of IM

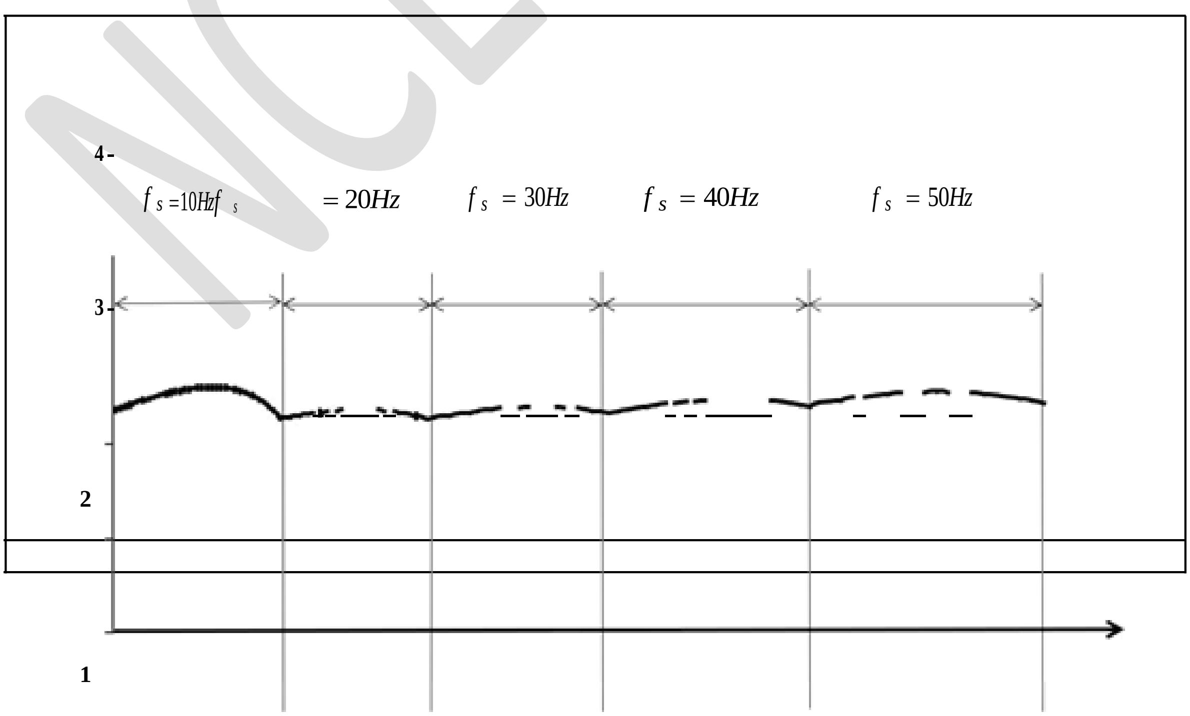
From **Figure 4** the following points can be observed:

- As the frequency of the stator input voltage increases, the maximum speed of the motor increases.
- With increase in frequency, the maximum torque produced by the motor also increases.
- The starting torque (torque at zero speed) does not vary much with increase in frequency.

In **Figure 5** the Torque vs. Speed curve for higher offset voltage $V_o = 7 i_{1n} r_{1n}$ is shown. It can be seen that using a higher offset voltage:

- the starting torque has increased.
- the maximum torque produced by the motor at different frequencies is almost constant.

Here the motor is operated at 10Hz between the rotor speeds ω_{r1} and ω_{r2}, at 20 Hz between ω_{r2} and ω_{r3} and so on. With this operation constant torque is maintained almost up to rated speed.



ω_{r1} ω_{r2} ω_{r3} ω_{r4} ω_{r5}

0

Speedpu

Figure 5: Constant Torque vs. Speed Curve for Constant volts/Hz control of IM

Utilizing the second point, a constant torque can be obtained from starting condition up to rated speed as shown in **Figure 6**.

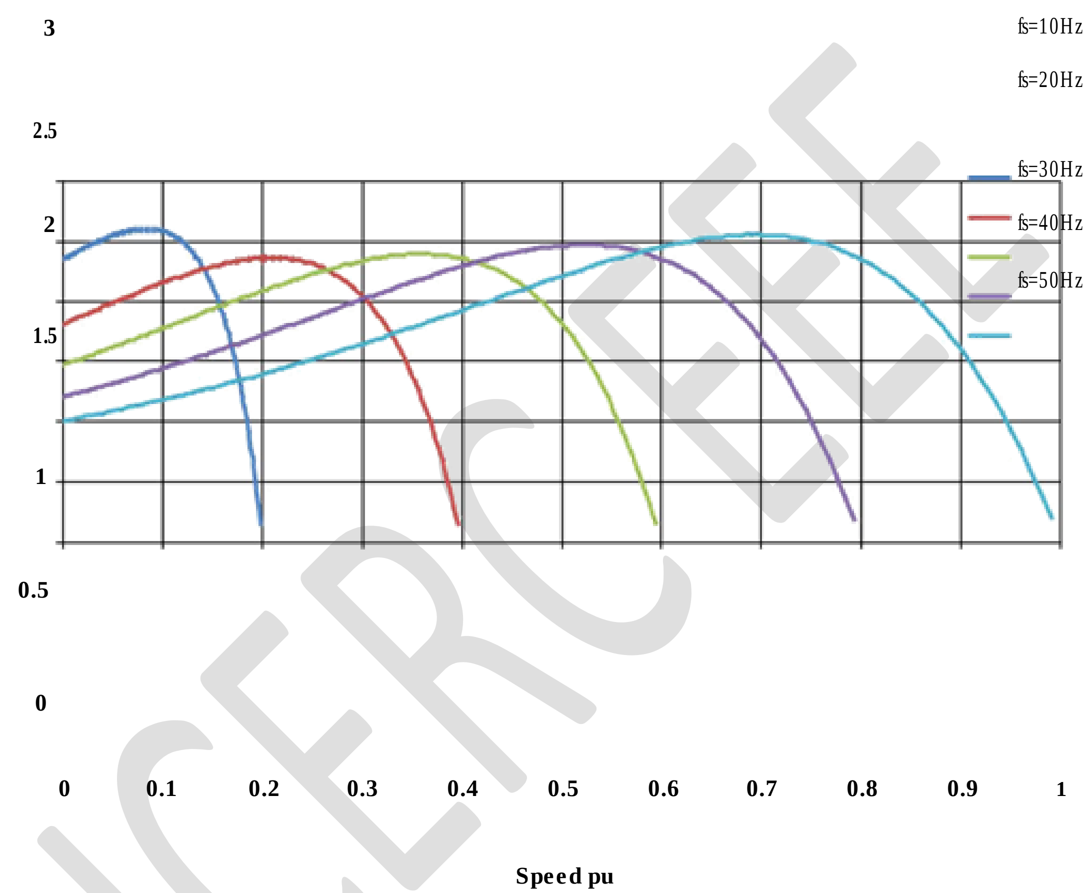


Figure 6: Torque vs. Speed Curve for Constant volts/Hz control of IM at higher offset voltage

The power factor vs. slip, stator current vs. slip curves and torque vs. slip for constant volts/Hz control are shown in **Figure 7**, **8** and **9** respectively. From **Figures 7-9** the following can be observed:

- As the slip increases (speed decreases) the power factor of the motor decreases. It attains a maximum value at a small slip (s_{pf}) value and then drops sharply.

- As the frequency increases the slop of the power factor between s_{pf} and unity slip increases.

- For any given slip, the magnitude of the stator current increases as the frequency increases. The magnitude of torque at a given slip also increases with increase in slip with the exception of unity slip (starting condition).

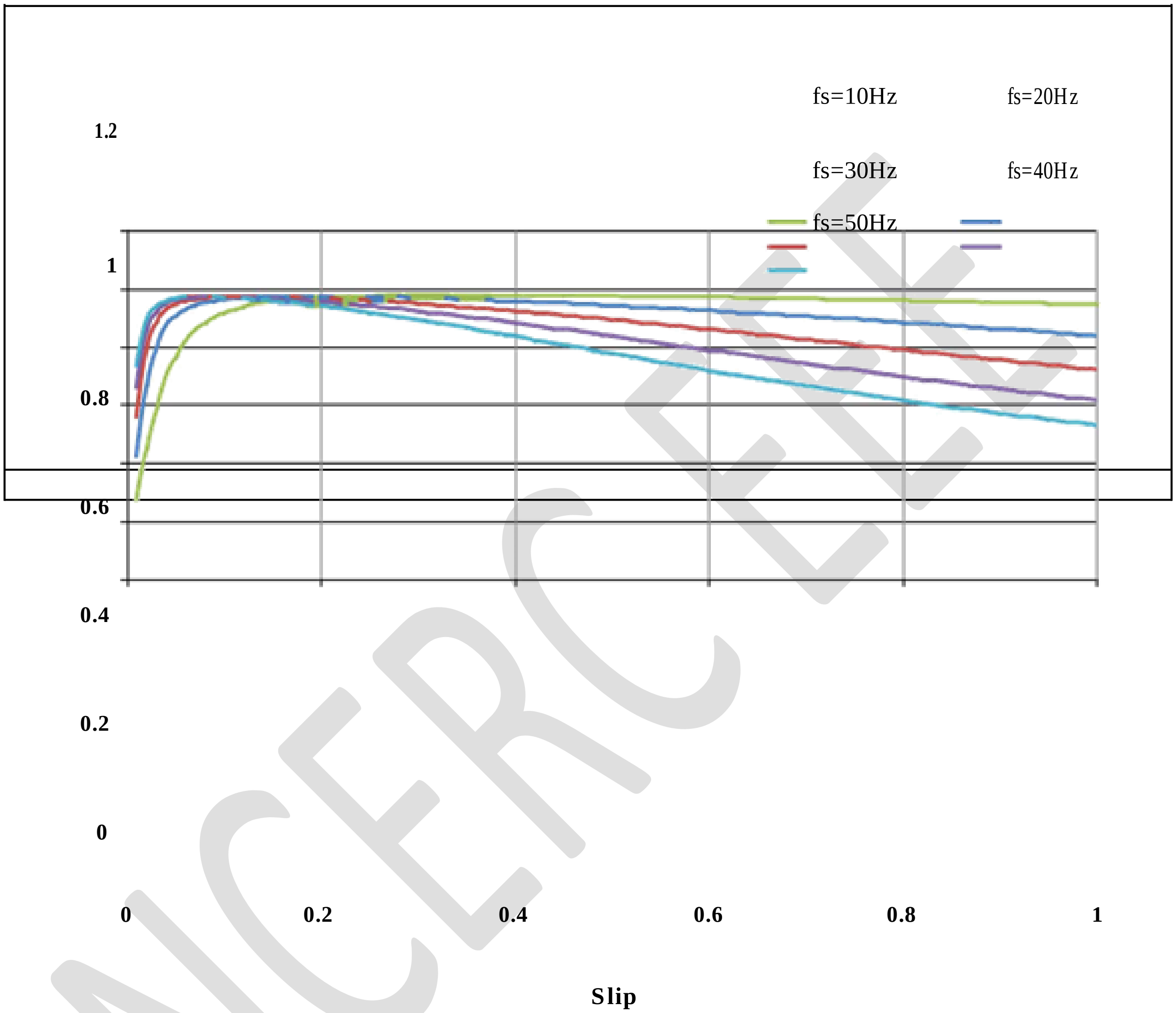


Figure 7: Power factor vs. Slip Curve for Constant volts/Hz control of IM

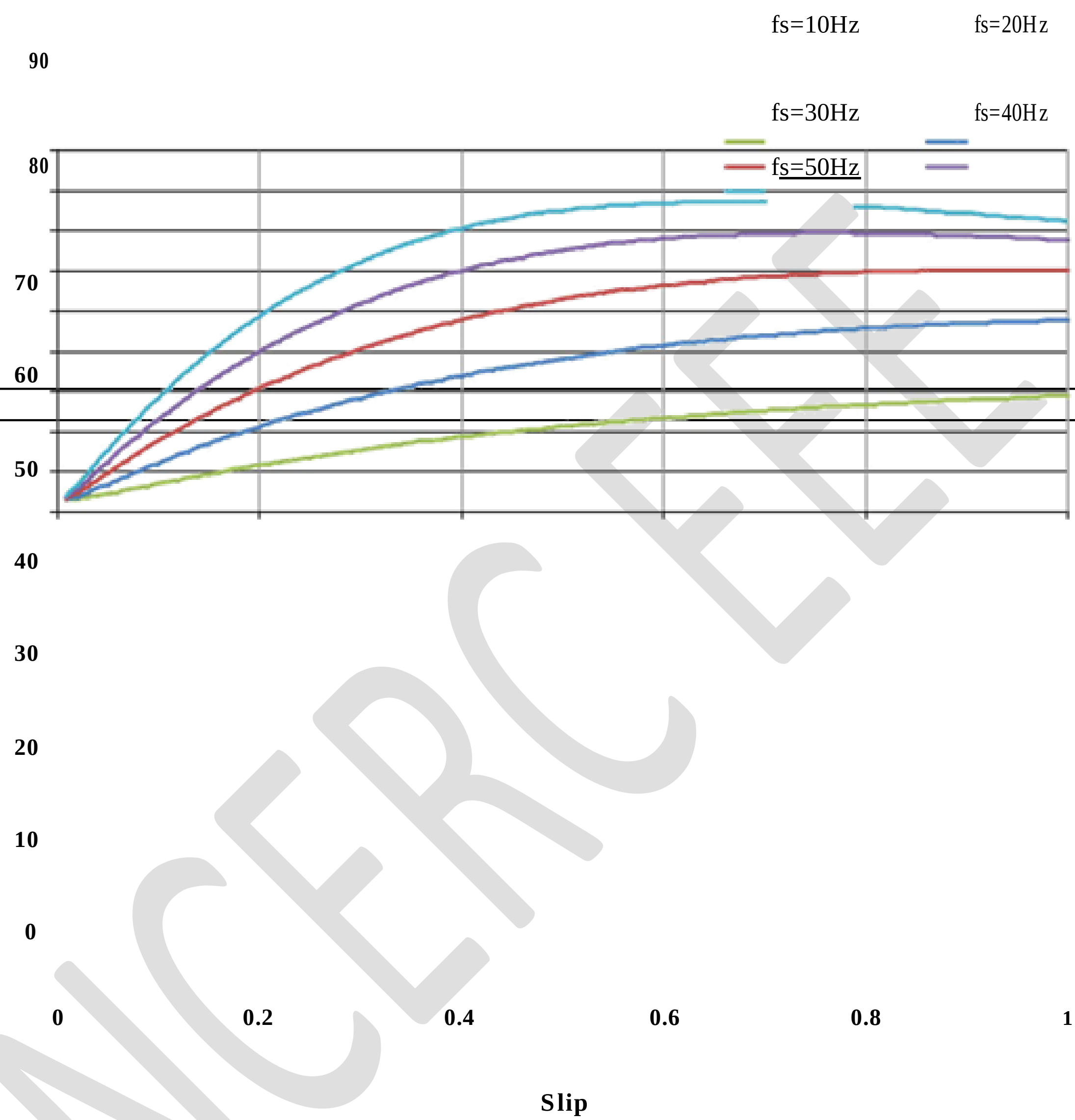


Figure 8: Stator Current vs. Slip Curve for Constant volts/Hz control of IM

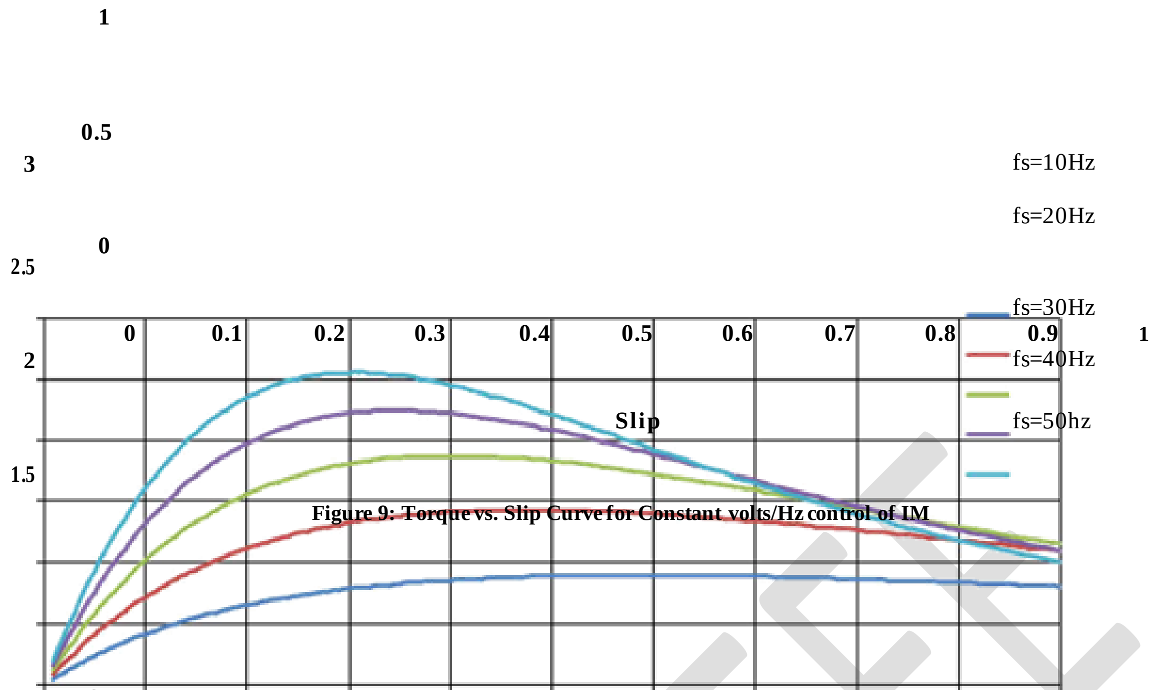


Figure 9: Torque vs. Slip Curve for Constant volts/Hz control of IM

References:

- R. Krishnan, *Electric motor drives: modeling, analysis, and control*, Prentice Hall, 2001

Lecture 20: Permanent magnet motor drives, their control and applications in EV/HEVs

Modeling of Induction Motor

Introduction

The topics covered in this chapter are as follows:

- Voltage Relations of Induction Motor
- Torque Equation in Machine Variables
- Equation of Transformation for Stator Variables
- Equation of Transformation for Rotor Variables
- Voltage and Torque Equations in Arbitrary Reference Frame Variables

Voltage Relations of Induction Motor

A 2 pole, 3 phase, Y connected symmetrical IM is shown in **Figure 1**. The stator windings are identical with N_s number of turns and the resistance of each phase winding is r_s . The rotor windings, may be wound or forged as squirrel cage winding, can be approximated as identical windings with equivalent turns N_r and resistance r_r . The air gap

of IM is uniform and the stator and rotor windings are assumed to be sinusoidally distributed. The sinusoidal distribution of the windings results in sinusoidal magnetic field in the air gap.

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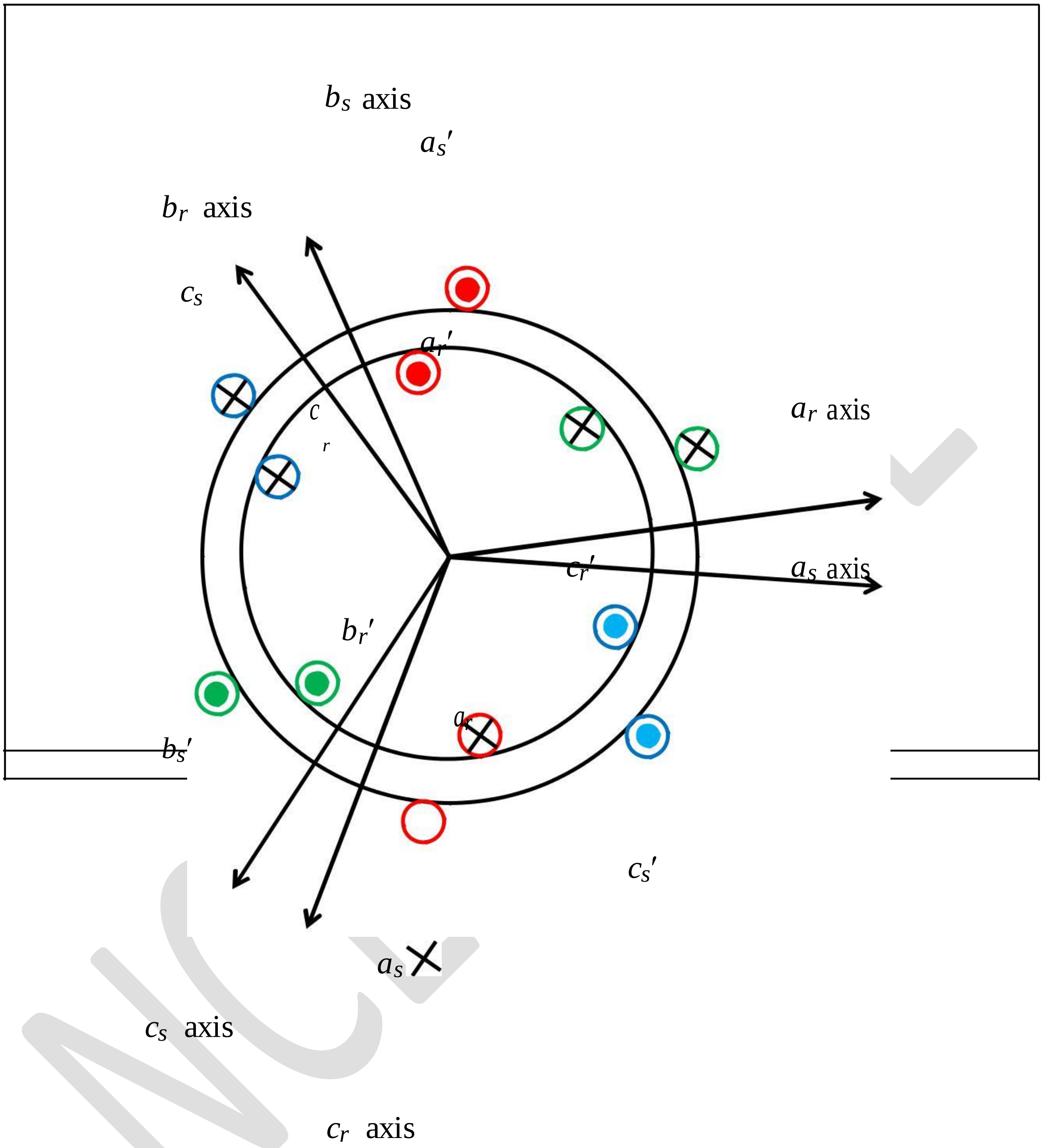

Figure 1: Two pole, 3 phase, wye-connected symmetrical induction motor

Since the windings are symmetric, the self inductances of stator windings are equal; that is

$$L_{asas} = L_{bsbs} = L_{cscs}$$

$$L_{asas} = L_{ls} + L_{ms}$$

where

L_{ls} is the leakage inductance of phase A or B or C of stator winding (1)

L_{ms} is the stator magnetizing inductance

L_{asas}, L_{bsbs}, L_{cscs} are the self inductances of stator phases

Similarly, the mutual inductances between the phases of the stator wind are same; that is

$$L_{asbs} = L_{bscs} = L_{csas} = -\frac{1}{2} L_{ms} \quad (2)$$

Based on the above discussion, the rotor self inductances and the mutual inductances between the rotor phases are

$$L_{arar} = L_{brbr} = L_{crcr}$$

$$L_{arar} = L_{lr} + L_{mr} \quad (3)$$

where

L_{lr} is the leakage inductance of phase a of rotor winding L_{mr} is the rotor magnetizing inductance

$$L_{arbr} = L_{brcr} = L_{crar} = -\frac{1}{2} L_{mr} \quad (4)$$

There exists mutual inductance between the stator and rotor windings. This mutual inductance is not constant because as the rotor rotates, the angle between the stator and rotor windings changes. Hence, the mutual inductance between the stator and rotor windings can be expressed as:

$$L_{asar} = L_{bsbr} = L_{cscr} = L_{sr}\cos(\theta_r)$$

$$L_{asbr} = L_{bscr} = L_{csar} = L_{sr}\cos\left(\theta_r + \frac{2\pi}{3}\right) \quad (5)$$

$$L_{ascr} = L_{bsar} = L_{csbr} = L_{sr}\cos\left(\theta_r - \frac{2\pi}{3}\right)$$

The voltage equations for the IM shown in **Figure 1** are

$$v_{as} = r_s i_{as} + \frac{d\lambda_{as}}{dt}$$

$$v_{bs} = r_s i_{bs} + \frac{d\lambda_{bs}}{dt}$$

$$v_{cs} = r_s i_{cs} + \frac{d\lambda_{cs}}{dt}$$

$$v_{ar} = r_r i_{ar} + \frac{d\lambda_{ar}}{dt} \qquad (6)$$

$$v_{br} = r_r i_{ar} + \frac{d\lambda_{ar}}{dt}$$

$$v_{cr} = r_r i_{cr} + \frac{d\lambda_{cr}}{dt}$$

The **equation 4** can be expressed in matrix form as

$$v_{abs} = r_s i_{abcs} + p\lambda_{abcs}$$
$$v_{abr} = r_r i_{abcr} + p\lambda_{abcr}$$

where

$$(v_{abcs})^T = [v_{as} \quad v_{bs} \quad v_{cs}]$$
$$(v_{abcr})^T = [v_{ar} \quad v_{br} \quad v_{cr}]$$ (7)
$$(i_{abcs})^T = [i_{as} \quad i_{bs} \quad i_{cs}]$$
$$(i_{abcr})^T = [i_{ar} \quad i_{br} \quad i_{cr}]$$
$$p = \frac{d}{dt}$$

In the above equation the subscript ***s*** refers to ***stator*** and ***r*** refers to ***rotor***. For a magnetically linear system, the flux linkages may be expressed as

$$\begin{bmatrix} \lambda_{abcs} \\ \lambda_{abcr} \end{bmatrix} = \begin{bmatrix} L_s & L_{sr} \\ (L_{sr})^T & L_r \end{bmatrix} \begin{bmatrix} i_{abcs} \\ i_{abcr} \end{bmatrix}$$ (8)

The stator and rotor windings inductances consist of self and mutual inductances and is represented as

$$L_s = \begin{bmatrix} L_{ls} + L_{ms} & -\frac{1}{2} L_{ms} & -\frac{1}{2} L_{ms} \\ -\frac{1}{2} L_{ms} & L_{ls} + L_{ms} & -\frac{1}{2} L_{ms} \\ -\frac{1}{2} L_{ms} & 1 & \end{bmatrix} \qquad L_r = \begin{bmatrix} L_{lr} + L_{mr} \\ -\frac{1}{2} L_{mr} \\ -\frac{1}{2} L_{mr} \end{bmatrix}$$

$$-\frac{1}{2}L_{ms} \quad L_{ls}+L_{ms}$$

$$(9)$$

$$\begin{matrix} -\frac{1}{2}L_{mr} & -\frac{1}{2}L_{mr} \\ L_{lr}+L_{mr} & -\frac{1}{2}L_{mr} \\ -\frac{1}{2}L_{mr} & L_{lr}+L_{mr} \end{matrix} \quad (10)$$

$$L_{sr} = L_{sr}\begin{bmatrix} \cos(\theta_r) & \cos(\theta_r+\frac{2\pi}{3}) & \cos(\theta_r-\frac{2\pi}{3}) \\ \cos(\theta_r-\frac{2\pi}{3}) & \cos(\theta_r) & \cos(\theta_r+\frac{2\pi}{3}) \\ \cos(\theta_r+\frac{2\pi}{3}) & \cos(\theta_r-\frac{2\pi}{3}) & \cos(\theta_r) \end{bmatrix} \quad (11)$$

The expression of voltage equation becomes convenient if all the rotor variables are referred to stator windings using the turns ratio:

$$i'_{abcr} = \frac{N_r}{N_s} i_{abcr}$$

$$v'_{abcr} = \frac{N_r}{N_s} v_{abcr} \tag{12}$$

$$\lambda'_{abcr} = \frac{N_r}{N_s} \lambda_{abcr}$$

The magnetizing and mutual inductances are associated with the same magnetic flux path; hence,

$$L_r' = \left(\frac{N_s}{N_r}\right)^2 L_r ; \quad L_{lr}' = \left(\frac{N_s}{N_r}\right)^2 L_{lr}$$

$$\therefore L_r' = \begin{bmatrix} L_{lr}' + L_{ms} & -\frac{1}{2}L_{ms} & -\frac{1}{2}L_{ms} \\ -\frac{1}{2}L_{ms} & L_{lr}' + L_{ms} & -\frac{1}{2}L_{ms} \\ -\frac{1}{2}L_{ms} & -\frac{1}{2}L_{ms} & L_{lr}' + L_{ms} \end{bmatrix} \tag{13}$$

Using **equation 13**, the flux linkages given by **equation 8** can be expressed as

$$\begin{bmatrix} \lambda_{abcs} \end{bmatrix} = \begin{bmatrix} L_s & L'_{sr} \\ \quad)^T \end{bmatrix}$$

$$\begin{bmatrix} \lambda'_{abcr} \end{bmatrix} \quad \begin{bmatrix} (L'_{sr})^T & L'_r \end{bmatrix} \begin{bmatrix} i'_{abcr} \end{bmatrix}$$

and using **equation 14**, the voltage equation (**equation 7) can be written as**

$$\begin{bmatrix} v_{abcs} \\ v'_{abcr} \end{bmatrix} = \begin{bmatrix} r_s + pL_s & pL'_{sr} \\ p(L'_{sr})^T & r'_r + pL'_r \end{bmatrix} \begin{bmatrix} i_{abcs} \\ i'_{abcr} \end{bmatrix} \qquad (14)$$

$$\text{where } r'_r = \left(\frac{N_s}{N_r}\right)^2 r_r \qquad (15)$$

Torque Equation in Machine Variables

The conversion into machine variables can be done using the principle of magnetic energy. In a machine, the stored magnetic energy is the sum of the self-inductance of each winding. The energy stored due to stator winding is:

$$W_s = \frac{1}{2}(i_{abcs})^T (L_s - L_{ls} I) i_{abc}$$

where (16)

I is the identity matrix

Similarly, the energy stored due to rotor winding is

$$W_r = \frac{1}{2}(i'_{abcr})^T (L'_r - L'_{lr} I) i'_{abcr} \qquad (17)$$

The energy stored due to mutual inductance between the stator and rotor windings is

$$W_{sr} = (i_{abcs})^T L'_{sr}\, i'_{abcr} \tag{18}$$

Hence, the total energy stored in the magnetic circuit of the motor is

$$W_f = W_s + W_{sr} + W_r$$

$$= \frac{1}{2}(i_{abcs})^T (L_s - L_{ls} I)\, i_{abcs} + (i_{abcs})^T L'_{sr}\, i'_{abcr} + \frac{1}{2}(i'_{abcr})^T (L'_r - L'_{lr} I)\, i'_{abcr} \tag{19}$$

Since the magnetic circuit of the machine is assumed to be linear (the of **B** vs. **H**), the stored energy in the magnetic field W_f is equal to the co-energy W_{co} . The electromagnetic torque produced by the IM is given by

$$T_e = \left(\frac{P}{2}\right)\frac{\partial W_{co}}{\partial \theta_r} = \left(\frac{P}{2}\right)\frac{\partial W_f}{\partial \theta_r} \tag{20}$$

where θ_r is the rotor angle at any given point of time.

Since the inductances L_s , L_{ls} , L'_r , L'_{lr} are not functions of θ_r and only L'_{sr} is a function of θ_r (**equation 11**), substituting the **equation 19** into **equation 21** gives

$$T_e = \left(\frac{P}{2}\right)(i_{abcs})^T \frac{\partial}{\partial \theta_r}\left[L'_{sr}\right] i'_{abcr} \tag{21}$$

Linear Transformations

From **equation 6** it can be seen that in order to study the dynamic behaviour of IM, a set of ***six*** equations have to be solved. If the number of equations to be solved is reduced, the computational burden will be reduced. In order to reduce the number of equations ***linear transformation*** is carried out. It is very common to use linear transformation to solve problems and one of the most common examples of it is ***logarithm***. The logarithms are used to multiply or divide two numbers. Similarly, the Laplace transform is also a linear transformation. It transforms the time-domain equations to s – domain equation and after manipulations, one again gets the required time-domain solution. The process of referring secondary quantities to primary or primary to secondary in a transformer is also equivalent to a linear transformation. It should be noted that ***the transformation from old to new set of variables is carried out for simplifying the calculations***.

Linear transformations in electrical machines are usually carried out to obtain new equations which are fewer in number or are more easily solved. For example, a three phase machine are more complicated because of the magnetic coupling amongst the three phase windings as seen from **equation 11**, but this is not the case after the transformation.

Transformation from Three Phases to Two Phases (a, b, c to α, β, 0)

A symmetrical 2 pole, 3 phase winding on the rotor is represented by three coils ***A, B, C*** each of N_r turns and displaced by 120° is shown in **Figure 2**.

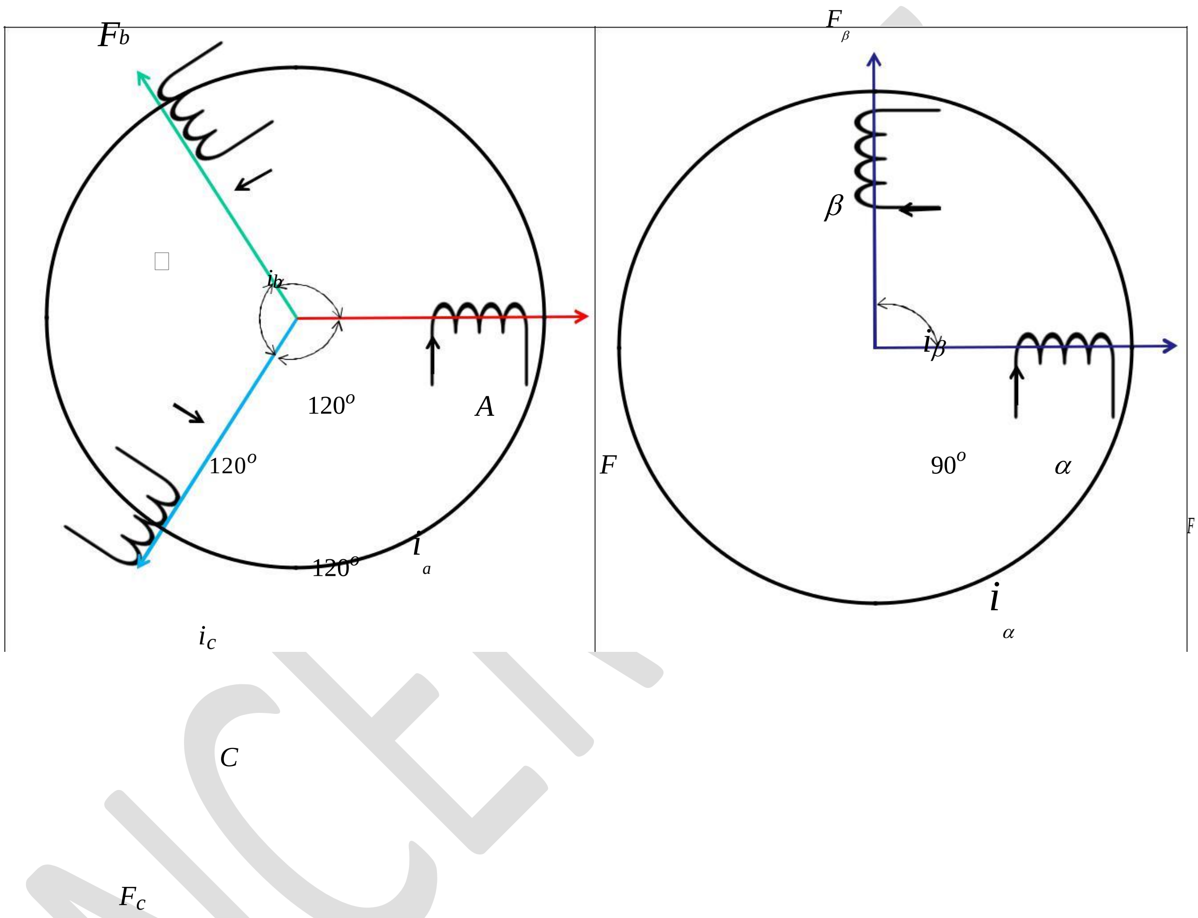

Figure 2: A symmetrical 2 pole 3phase winding on the rotor **Figure 3: A balanced 2phase winding of the rotor**

Maximum values of mmfs F_a , F_b , F_c are shown along their respective phase axes. The combined effect of these three mmfs results in mmf of constant magnitude rotating at a constant angular velocity depending on the poles and frequency. If the three currents in the rotor are:

$$i_a = I_m \cos(\omega t);\ i_b = I_m \cos\left(\omega t - \frac{2\pi}{\ }\right);\ i_c = I_m \cos\left(\omega t - \frac{4\pi}{\ }\right) \quad (22)$$

$\Big(\quad 3 \Big) \qquad \Big(\quad 3 \Big)$

The currents given in **equation 22** produce a mmf of constant magnitude $\frac{3I_m N_r}{2}$ rotating with respect to three phase winding at the frequency of ω. In **Figure 3**, a balanced two phase winding is represented by two orthogonal coils α , β on the rotor. For the sake of convenience the axes of phase ***A*** and α are taken to be coincident. The two phase currents flowing in the winding is given by

$$i_\alpha = I_m \cos(\omega t);\ i_\beta = I_m \cos\left(\omega t - \frac{\pi}{2}\right) \qquad (23)$$

These two phase currents result in a mmf of constant magnitude $I_m N_r$ rotating with respect to the two phase windings at the frequency of the currents. The mmf of three phase and two phase systems can be rendered equal in magnitude by making any one of the following changes:

- By changing the magnitude of the two phase currents
- By changing the number of turns of the two phase windings
- By changing both the magnitude of currents and number of turns

In the following subsections each of the three cases are discussed.

Changing the magnitude of two phase currents

In this case the number of turns in the two phase winding is N_r which is same as that of the three phase windings. Hence, in order to have equal mmf, the new magnitude of the current in the two phases must be determined. To obtain the new values of the two phase currents the instantaneous three phase mmfs are resolved along the α axis shown in **Figure 3**:

$$i_\alpha N_r = \left(i_a \cos 0 + i_b \cos\frac{2\pi}{3} + i_c \cos\frac{4\pi}{3} \right) N_r \Rightarrow i_\alpha = i_a - \frac{1}{2}\left(i_b + i_c\right) \qquad (24)$$

Similarly, the resolving the three phase currents along the β axis gives

$$i_\beta N_r = \left(i_a \sin 0 + i_b \sin\frac{2\pi}{3} + i_c \sin\frac{4\pi}{3} \right) N_r \Rightarrow i_\beta = \left(\frac{\sqrt{3}}{2} i_b - \frac{\sqrt{3}}{2} i_c \right) \qquad (25)$$

For a balanced three phase system the sum of three currents is zero, that is

$$i_a + i_b + i_c = 0 \qquad (26)$$

Using **equation 26** into **equation 24** gives

$$i_\alpha = \frac{3}{2} i_a \tag{27}$$

Substituting the values of i_a , i_b and i_c from **equation 22** into **equations 25** and **27** gives

$$i_\alpha = \frac{3}{2} I_m \cos(\omega t);\ i_\beta = \frac{3}{2} I_m \sin(\omega t) \tag{28}$$

From **equation28** it can be seen that the magnitude of the two phase currents is 3/2 times the magnitude of the three phase currents. Since the number of turns per phase is same in both the three and two phase windings, the magnitude of phase e.m.fs of the two and three phase windings would be equal. The power per phase of the two phase system is $3/2VI_m$ and the power per phase of a three phase winding is VI_m . However, the total power produced by a two phase system is ($= 2 \cdot 3/2 \cdot VI_m = 3VI_m$) and that produced by a three phase system is $3VI_m$. Thus, the linear transformation is power invariant. The only disadvantage is that the transformation of current and voltage will differ because of presence of factor 3/2 in the current transformation. As factor 3/2 appears in current transformation and not in voltage transformation, the per phase parameters of the two phase and three phase machine will not be the same.

Changing the number of turns of two phase winding

If the number of turns of two phase winding is made 3/2 times that of the three phase winding, then for equal mmfs the following relation between the two phase and three phase currents holds:

$$\frac{3}{2} i_\alpha N_r = \left(i_a \cos 0 + i_b \cos\frac{2\pi}{3} + i_c \cos\frac{4\pi}{3} \right) N_r \Rightarrow \frac{3}{2} i_\alpha = i_a - \frac{1}{2}\left(i_b + i_c\right) = \frac{3}{2} i_a \Rightarrow i_\alpha = i_a \qquad (29)$$

$$\frac{3}{2} i_\beta N_r = \left(i_a \sin 0 + i_b \sin\frac{2\pi}{3} + i_c \sin\frac{4\pi}{3} \right) N_r \Rightarrow i_\beta = \frac{2}{3}\left(\frac{\sqrt{3}}{2} i_b - \frac{\sqrt{3}}{2} i_c \right) \qquad (30)$$

Substituting the values of i_a , i_b and i_c from **equation 22** into **equations 29** and **30** gives

$$i_\alpha = I_m \cos\left(\omega t\right); i_\beta = I_m \sin\left(\omega t\right) \qquad (31)$$

Since, the number of turns in the two phase winding is 3/2 times that of three phase winding, the per phase voltage of the two phase machine will be 3/2 times the per phase voltage of the three phase systems. Hence, The power per phase in two phase system= $\frac{3}{2} VI_m$

Total in two phase system= $3VI_m$

The power per phase in three phase system= VI_m

Total in three phase system= $3VI_m$

Here again the power invariance is obtained, but, as in the previous case, the transformation of current and voltage will differ because of the factor 3/2 in the voltage transformation. In this case the per phase parameters of the machine will be different for two and three phase systems.

Changing both the number of turns and magnitude of current of two phase winding

In this case both the magnitude of currents and number of turns of the two phase system are changed to obtain identical transformation for voltage and current. To do so the number of turns in the two phase winding is made $\sqrt{\frac{3}{2}}$ times that of three phase winding. Then for equal m.m.f the following holds

$$\sqrt{\frac{3}{2}}\, i_\alpha N_r = \left(i_a \cos 0 + i_b \cos\frac{2\pi}{3} + i_c \cos\frac{4\pi}{3} \right) N_r$$

$$\Rightarrow i_\alpha = \sqrt{\frac{2}{3}} \left(i_a - \frac{1}{2} i_b - \frac{1}{2} i_c \right) \Rightarrow i_\alpha = \sqrt{\frac{3}{2}}\, I_m \cos(\omega t)$$

(32)

$$\sqrt{\frac{3}{2}}\, i_\beta N_r = \left(i_a \sin 0 + i_b \sin\frac{2\pi}{3} + i_c \sin\frac{4\pi}{3} \right) N_r$$

$$\Rightarrow i_\beta = \sqrt{\frac{2}{3}} \left(0 + \frac{\sqrt{3}}{2} i_b - \frac{\sqrt{3}}{2} i_c \right) \Rightarrow i_\beta = \sqrt{\frac{3}{2}}\, I_m \sin(\omega t) \qquad (33)$$

Since the number of turns in the two phase winding is $\sqrt{\frac{3}{2}}$ times that of three phase winding, the voltage per phase of the two phase winding is $\sqrt{\frac{3}{2}}$ times that of the three phase winding. Hence, the phase voltage and current of the two phase system are $\frac{\sqrt{3}}{2}$ times that of three phase system. This results in identical transformations for both the voltage and current and the per phase quantities of the machine, such as the impedance per phase, will be same for two and three phase systems.

Hence, the transformation equations for converting three phase currents into two phase currents, given by **equations 32** and **33**, can be expressed in matrix form as

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \tag{34}$$

The transformation matrix in **equation 34** is singular and hence i_a , i_b and i_c cannot be obtained from i_α , i_β . The matrix can be made square matrix if a third equation of constraint between i_a , i_b and i_c is introduced. Since, the magnitude and direction of the mmf produced by two and three phase systems are identical, the third current in terms of i_a , i_b and i_c should not produce any resultant air gap mmf. Hence, a zero sequence current is introduced and it is given by

$$i_0 = \frac{1}{\sqrt{3}}(i_a + i_b + i_c) \qquad (35)$$

Due to the fact that sum of three phase currents in a balanced system is zero (**equation 26**), the zero sequence current does not produce any rotating mmf. Using the **equation 35** the matrix representation given in **equation 34** can be written as

$$\begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \qquad (36)$$

The transformation matrix now is non-singular and its inverse can be easily obtained.

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Transformation from Rotating Axes ($\alpha, \beta, 0$) to Stationary Axes ($d, q, 0$)

In **Figure 3**, three phase and two phase windings are shown on the rotor and hence both $d, q, 0$ the windings rotate at the same speed and in the same direction. Hence, both the two phase and three phase windings are at rest with respect to each other.

In this section the transformation of rotating $\alpha, \beta, 0$ quantities to stationary

quantities is carried out. When transformation is carried out from rotating to stationary axes, the relative position of rotating axes varies with respect to stationary or fixed axes. Hence, the transformation matrix must have coefficients that are functions of the relative position of the moving α, β and fixed d, q axes. In **Figure 4** the rotating α, β axes are shown inside the circle and the stationary d, q axes are shown outside. The angle θ_r shown in **Figure 4** is such that at time $t = 0$, $\theta_r = 0$, that is, the α, β axis is aligned with the d, q axis.

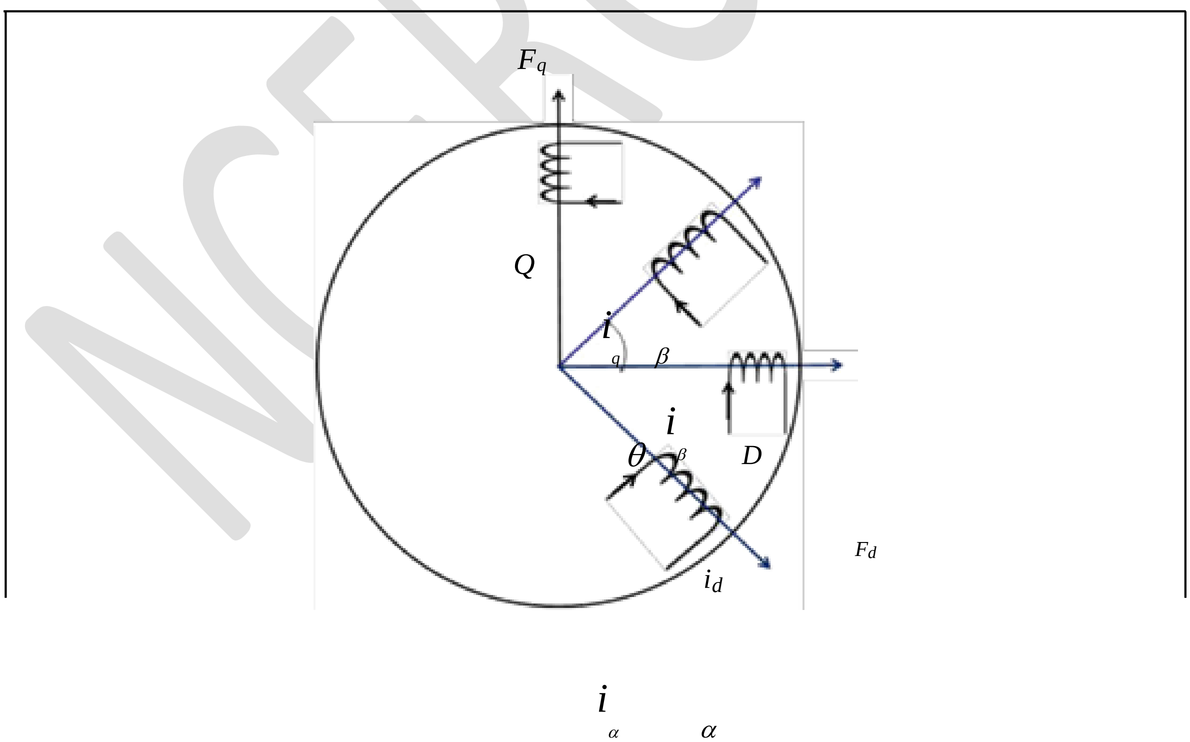


Figure 4: The d and q axis on the rotor

At any time t , $\theta_r = \omega_r t$, where ω_r is the angular speed of the rotor. Assuming same d , number of turns in the α , β and q windings, the mmfs F_α and F_β can be resolved along the d , q axis as

$$F_d = F_\alpha \cos\theta_r + F_\beta \sin\theta_r \Rightarrow N_r i_d = N_r i_\alpha \cos\theta_r + N_r i_\beta \sin\theta_r$$

$$\Rightarrow i_d = i_\alpha \cos\theta_r + i_\beta \sin\theta_r \qquad (37)$$

$$i_q = -i_\alpha \sin\theta_r + i_\beta \cos\theta_r$$

The **equation** 37 can be expressed in the matrix form as

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} \cos\theta_r & \sin\theta_r \\ -\sin\theta_r & \cos\theta_r \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix}$$

and (38)

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \begin{bmatrix} \cos\theta_r & -\sin\theta_r \\ \sin\theta_r & \cos\theta_r \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix}$$

Let the currents in d , q axis winding be

$$i_d = I_m \sin\left(\omega t + \phi\right)$$

$$i_q = I_m \cos\left(\omega t + \phi\right) \qquad (39)$$

where

- is a constant arbitrary phase angle

Using **equation 38** and **equation 39**, the currents i_α , i_β are obtained as

$$i_\alpha = I_m \sin\left(\theta - \theta_r + \phi\right)$$

$$i_\beta = I_m \cos\left(\theta - \theta_r + \phi\right)$$

- $i_\alpha = I_m \sin\left(\omega t - \omega_r t + \phi\right) \left(\omega t - \omega_r t + \phi\right) i_\beta = I_m \cos$ (40)

where

$\varpi\iota. \quad = \omega t$

In case the frequency of the d and q axis current is same as the speed of rotation of the rotor, then

$$i_\alpha = I_m \sin\left(\phi\right)$$

(41)

$$i_\beta = I_m \cos\left(\phi\right)$$

Thus, time varying currents in stationary d , q axis result in mmf which is identical to the mmf produced by constant currents (or d.c.) in rotating α , β axis.

In the above transformation the zero sequence current is not transformed and it can be taken into account by an additional column in the **equation 38**.

$$\begin{bmatrix} i_d \\ i_q \\ i_o \end{bmatrix} = \begin{bmatrix} \cos\theta_r & \sin\theta_r & 0 \\ -\sin\theta_r & \cos\theta_r & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \\ i_o \end{bmatrix} \qquad (42)$$

Substituting the values of i_α, i_β, i_0 from **equation 32** into **equation 42** gives

$$\begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} = \frac{2}{3}\begin{bmatrix} \cos\theta_r & \cos\left(\theta_r - \frac{2\pi}{3}\right) & \cos\left(\theta_r + \frac{2\pi}{3}\right) \\ -\sin\theta_r & -\sin\left(\theta_r - \frac{2\pi}{3}\right) & -\sin\left(\theta_r + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

$$\left(i_d \quad i_q \quad i_0\right)^T = K_s\left(i_a \quad i_b \quad i_c\right)^T \tag{43}$$

where

$$K_s = \frac{2}{3}\begin{bmatrix} \cos\theta_r & \cos\left(\theta_r - \frac{2\pi}{3}\right) & \cos\left(\theta_r + \frac{2\pi}{3}\right) \\ \sin\theta_r & \sin\left(\theta_r - \frac{2\pi}{3}\right) & \sin\left(\theta_r + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

The inverse transformation matrix is given by

$$\begin{bmatrix} \cos\theta_r & -\sin\theta_r & \frac{1}{2} \\ \end{bmatrix}$$

$$K_S^{-1} = \sqrt{\frac{2}{3}}\begin{bmatrix} \cos\left(\theta_r - \frac{2\pi}{3}\right) & -\sin\left(\theta_r - \frac{2\pi}{3}\right) & \frac{1}{\sqrt{2}} \\ \cos\left(\theta_r + \frac{2\pi}{3}\right) & -\sin\left(\theta_r + \frac{2\pi}{3}\right) & \frac{1}{\sqrt{2}} \end{bmatrix} \tag{44}$$

The above transformation is valid for any electrical quantity such as current, voltage, flux linkage, etc. In general the three phase voltages, currents and fluxes can be converted into *dqo* phases using the following transformation matrices

$$\Psi \quad (f_{qdo})^T = K_s (f_{abc})^T \tag{45}$$

where

$$(f_{qdo})^T = \begin{bmatrix} f_q & f_d & f_o \end{bmatrix}; \quad (f_{abc})^T = \begin{bmatrix} f_a & f_b & f_c \end{bmatrix}$$

Transformation of Induction Motor Quantities into Stationary d, q Axis

The three phase stator and rotor wind axis are shown in **Figure 5a**. In **Figure 5a**, the subscript ***s*** represents stator quantities and ***r*** represents rotor quantities. For the stator

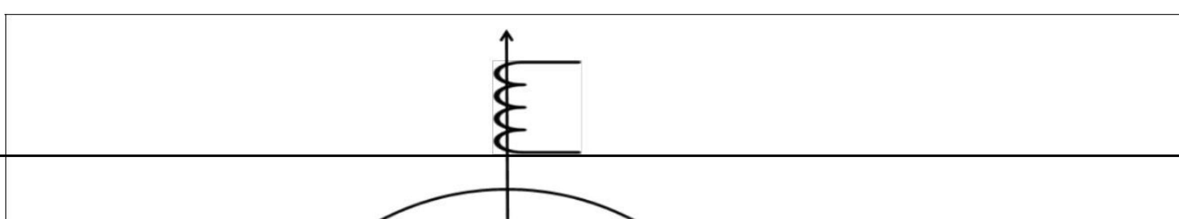
δ, β axis and d, q axis are coincident as shown in **Figure 5b** and hence there is no difference between α, β and d, q stator quantities. Examination of **Figure 5a** and **5b** reveals that phase A_s coincides with the phase α axis or phase d axis of the 2 phase

machines. As a result of this, the results obtained for d axis quantities apply equally well to the α phase of the 2 phase machine. The conversion of 3 phase stator winding to 2 phase stator winding is given by **equation 36** as

$$\begin{bmatrix} i_{\alpha s} \\ i_{\beta s} \\ i_{0s} \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} \qquad (46)$$

Since the α, β and d, q axis both lie on the stator are stationary with respect to each other, the transformation from α, β to d, q axis is given by

$$\begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} i_{\alpha s} \\ i_{\beta s} \end{bmatrix} \qquad (47)$$



[11] s

β_r

α_s

α_r

Figure 5a: Equivalent 2 phase induction machine in α , β axis

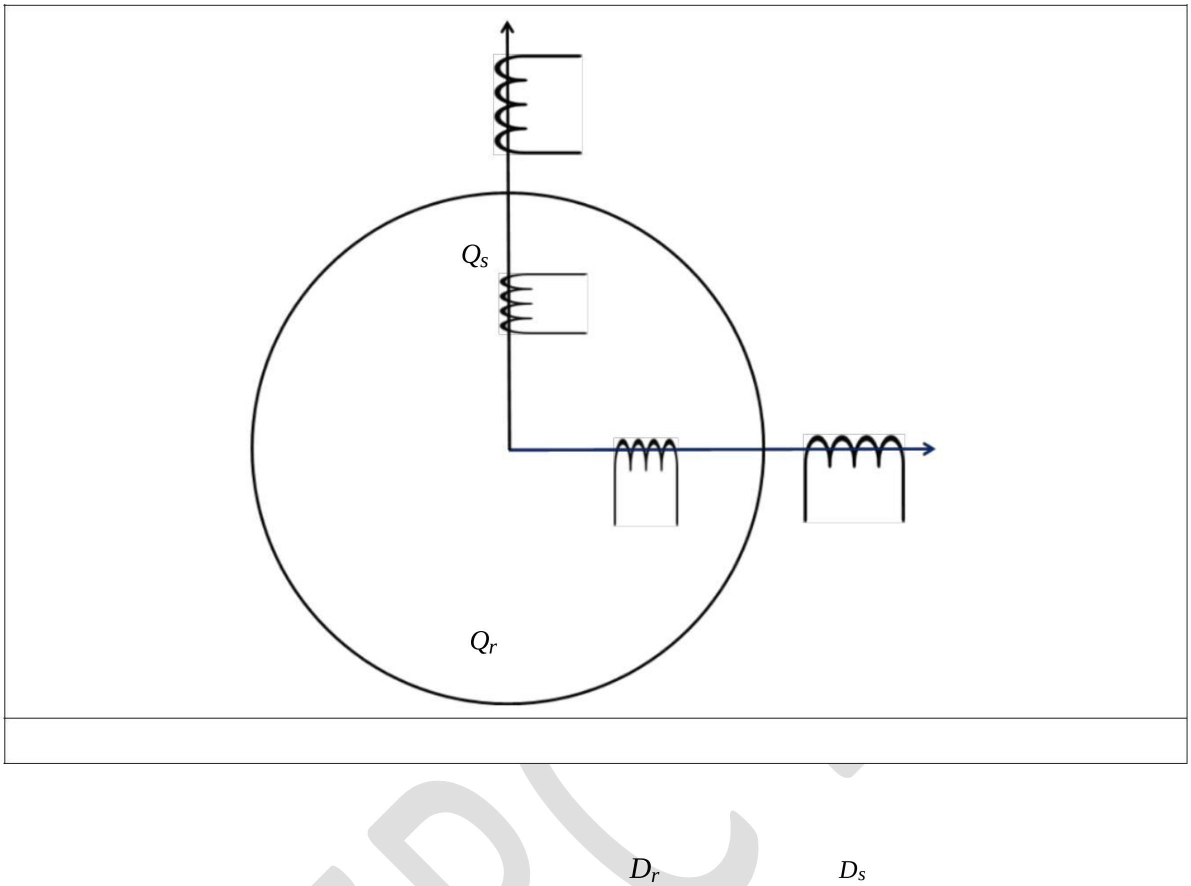


Figure 5b: Equivalent 2 phase induction machine in d , q axis

In case of rotor currents, the transformation from 3 phase to 2 phase is given by

$$\begin{bmatrix} i_{\alpha r} \\ i_{\beta r} \\ i_{0r} \end{bmatrix}$$

$$\begin{bmatrix} i_{\alpha r} \\ i_{\beta r} \\ i_{0r} \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} i_{ar} \\ i_{br} \\ i_{cr} \end{bmatrix} \qquad (48)$$

The transformation 2 phase rotor currents from rotating α, β to stationary d, q axis is given by

$$\begin{bmatrix} i_{dr} \\ i_{qr} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} i_{\alpha r} \\ i_{\beta r} \end{bmatrix} \qquad (49)$$

The rotor voltages are related by identical expressions. The frequency of the currents in 3phase rotor windings or in 2phase rotor windings is the slip frequency sf. Since the d, q axes windings are stationary, the frequency of rotor currents i_{dr} and i_{qr} is the line frequency f.

The Machine Performance Equations

The general voltage equation for the machine shown in **Figure 5b** is

$$\begin{bmatrix} v_{ds} \\ v_{qs} \\ v_{dr} \\ v_{qr} \end{bmatrix} = \begin{bmatrix} r_{ds} + L_{ds}p & 0 & M_d p & 0 \\ 0 & r_{qs} + L_{qs}p & 0 & M_q p \\ M_d p & -M_q \omega_r & r_{dr} + L_{dr}p & -L_{qr}\omega_r \\ M_d \omega_r & M_q p & L_{dr}\omega_r & r_{qr} + L_{qr}p \end{bmatrix} \begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr} \end{bmatrix} \tag{50}$$

where $p = \frac{d}{dt}$

The following points about the IM are to be considered:

- Stator and rotor both have balanced winding configurations, hence: $r_{ds} = r_{qs} = r_s$ =resistance of each stator coil

  $r_{dr} = r_{qr} = r_r$ =resistance of each rotor coil

- Since the air gap is uniform, the self inductances of d and q axis of the stator winding are equal and that of the rotor windings are also equal, that is

  $L_{ds} = L_{qs} = L_s$ =self inductance of the stator winding

  $L_{dr} = L_{qr} = L_r$ =self inductance of the rotor winding

- The d and q axis coils are identical, the mutual inductance between the stator and rotor d axis coils is equal to the mutual inductance between stator and rotor q axis coils, that is

  $M_d = M_q = L_m$

In an induction machine the rotor windings are short circuited, therefore no emf exists in the winding of the rotor and $v_{dr} = v_{qr} = 0$. Since the rotor winding is short circuited, the direction of i_{dr} and i_{qr} are reversed and this has to be taken into account in the general voltage equations.

Based on the above discussion, the general voltage equation becomes:

$$\begin{bmatrix} v_{ds} \\ v_{qs} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r_s + L_s p & 0 & -L_m p & 0 \\ 0 & r_s + L_s p & 0 & -L_m p \\ L_m p & -L_m \omega_r & -\left(r_r + L_r p \right) & L_r \omega_r \\ L_m \omega_r & L_m p & -L_r \omega_r & -\left(r_r + L_r p \right) \end{bmatrix} \begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr} \end{bmatrix} \qquad (51)$$

The voltage equations given in **equation 51** can be written as:

$V = [R]I + [L]pI + [G]\omega_r I$ where

$$V = \begin{bmatrix} v_{ds} \\ v_{qs} \\ v'_{dr} \\ v'_{qr} \end{bmatrix}; \quad I = \begin{bmatrix} i_{ds} \\ i_{qs} \\ i'_{dr} \\ i'_{qr} \end{bmatrix}; \quad [R] = \begin{bmatrix} r_s & 0 & 0 & 0 \\ 0 & r_s & 0 & 0 \\ 0 & 0 & r_s & 0 \\ 0 & 0 & 0 & r_s \end{bmatrix} \tag{52}$$

$$[L] = \begin{bmatrix} L_s & 0 & L_m & 0 \\ 0 & L_s & 0 & L_m \\ L_m & 0 & L_r & 0 \\ 0 & L_m & 0 & L_r \end{bmatrix}; \quad [G] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -L_m & 0 & L_r \\ L_m & 0 & -L_r & 0 \end{bmatrix}$$

The instantaneous input power to the machine is given by

$$P = I^T V = I^T [R] I + I^T [L] pI + I^T [G] \omega_r I \tag{53}$$

In **equation 53** the term $I^T[R]I$ represents the stator and rotor resistive losses. The term $I^T[L]pI$ denotes the rate of change of stored magnetic energy. Hence, what is left of the power component must be equal to the air gap power given by the term $I^T[G]\omega_r I$. The air gap torque is given by

$$\omega_m T_e = I^T [G] \omega_r I$$

N

$$\omega_r = \frac{P}{2} \omega_m$$

- T_e

$$= \frac{N_p}{2} I^T [G] I \tag{54}$$

where

ω_m is the mechanical speed of the rotor

and N_p is the number of poles

Substituting the value of $[G]$ from **equation52** into **equation 54** gives

$$T_e = \frac{N_p}{2} L_m \left(i_{qs}\, i_{dr} - i_{ds}\, i_{qr} \right) \tag{55}$$

The torque given by **equation 55** can also be written as

$$T_e = \frac{N_p}{2} L_m \left(\lambda_{qr}\, i_{dr} - \lambda_{dr}\, i_{qr} \right) \tag{56}$$

.

References:

4 R. Krishnan, *Electric motor drives: modeling, analysis, and control*, Prentice Hall, 2001

5 P. C. Krause, O. Wasynczuk, S. D. Sudhoff, *Analysis of electric machinery*, IEEE Press, 1995

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Lecture 21: Switch reluctance motors, their configurations and optimization

Field Oriented Control of Induction Motor

Introduction

The topics covered in this chapter are as follows:

- ⇒ Field Oriented Control (FOC)
- ⇒ Direct Rotor Oriented FOC
- ⇒ Indirect Rotor Oriented FOC

Field Oriented Control (FOC)

In an Electric Vehicle, it is required that the traction motor is able to deliver the required torque almost instantaneously. In an induction motor (IM) drive, such performance can be achieved using a class of algorithms known as ***Field Oriented Control*** *(FOC)*. There are varieties of FOC such as:

- ⇒ Stator flux oriented
- ⇒ Rotor flux oriented
- ⇒ Air gap flux oriented

Each of the above mentioned control method can be implemented using ***direct*** or ***indirect*** methods.

The basic premise of FOC may be understood by considering the current loop in a uniform magnetic field as shown in **Figure 1a**. From Lorenz force equation, it can be seen that the torque acting on the current loop is given by

$$T_e = -2\,BiNLr\sin\theta$$

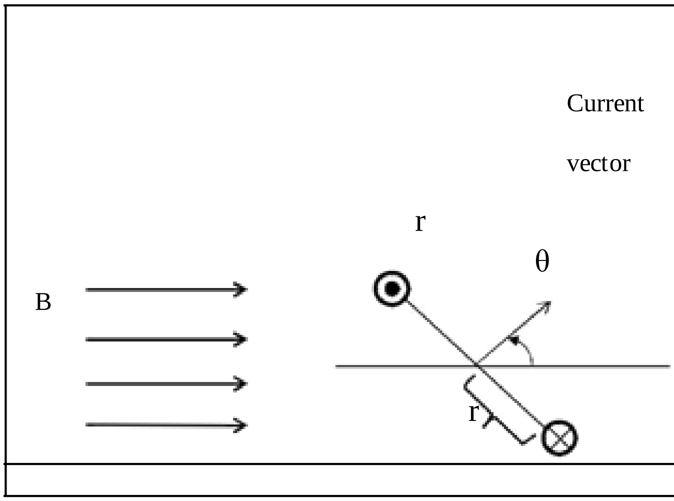
where

B is the flux density

i is the current (1)

□ is the number of turns
L is the length of the coil
r is the radius of the coil

.



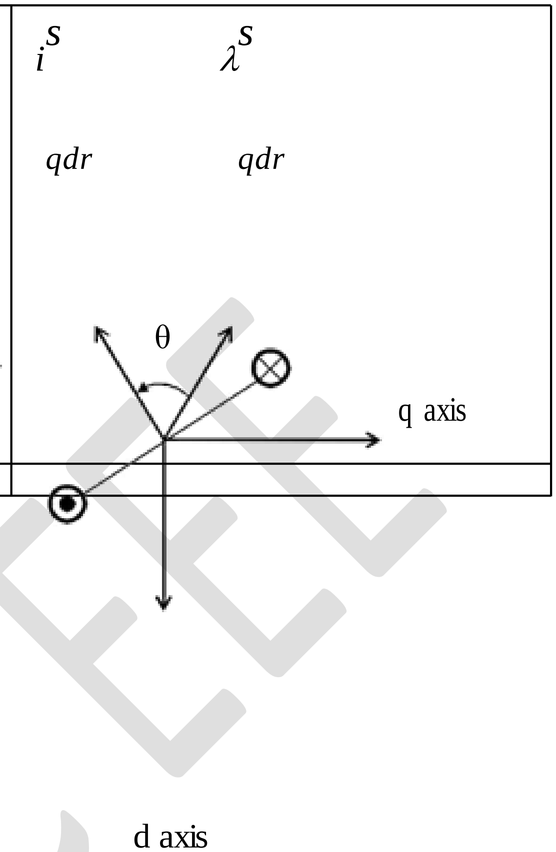


Figure 1a: A coil in a magnetic field **Figure 1b: Orientation of magnetic field in IM**

From **equation 1** it is evident that the torque is maximised when the current vector is perpendicular to the magnetic field. The same conclusion can be applied to an IM. In **Figure 1b** orientations of magnetic fields and currents in an IM are shown. The rotor current and flux linkage vectors are shown in **Figure 1** at some instant of time. Hence, the torque produced by the motor (refer to Lecture 19) is given by

$$T_e = \frac{3}{2}\frac{P}{2}\left(\lambda'_{qr} i'_{dr} - \lambda'_{dr} i'_{qr}\right) \qquad (2)$$

The **equation 2** can be re-written as

$$T_e = -\frac{3}{2}\frac{P}{2}\left|\lambda'_{qdr}\right|\left|i'_{qdr}\right|\sin\theta \qquad (3)$$

The **equation 3** is analogous to **equation 1**. Hence, for a given magnitude of flux linkage, torque is maximised when the flux linkage and current vectors are perpendicular. Therefore, it is desirable to keep the rotor flux linkage perpendicular to rotor current vector.

In the analysis of FOC the following convention will be used:

- The parameters with a superscript " s " are in stator frame of reference.
- The parameters with a superscript " e " are in synchronous frame of reference.
- The parameters with subscript " r " indicate rotor parameters.
- The parameters with subscript " s " indicate stator parameters.
- All rotor quantities are referred to stator using the turns ratio of the windings (Lecture 17) and hence " ' " is dropped.

In case of singly excited IMs (*in singly excited IM, the rotor winding is not fed by any external voltage source. In case of wound rotor machines, they are short circuited using slip rings. For cage IMs, the rotor bars are short circuited at the terminals*), the rotor flux linkage vector and rotor current vector are always perpendicular. The voltage equations for the IM (refer to Lecture 19) in synchronous frame of reference are

$$v_{qs}^e = r_s i_{qs}^e + \omega_e \lambda_{ds}^e + p\lambda_{qs}^e$$

$$v_{ds}^e = r_s i_{ds}^e - \omega_e \lambda_{qs}^e + p\lambda_{ds}^e$$

$$v_{os}^e = r_s i_{os}^e + p\lambda_{os}^e$$

$$v_{qr}^e = r_r i_{qr}^e + (\omega_e - \omega_r)\lambda_{ar}^e + p\lambda_{qr}^e$$

$$v_{dr}^e = r_r i_{dr}^e - (\omega_e - \omega_r)\lambda_{q}^e + p\lambda_{dr}^e \qquad (1)$$

$$v_{or}^e = r_r i_{or}^e + p\lambda_{or}^e$$

where

ω_e is the rotational speed of synchronous frame of reference

In case of singly excited IM, the rotor voltages are zero, that is $v_{qr}^e = 0$, $v_{dr}^e = 0$ and $v_{or}^e = 0$.

Hence, the rotor currents can be obtained as

$$0 = r_r i_{qr}^e + (\omega_e - \omega_r)\lambda_{dr}^e + p\lambda_{qr}^e \Rightarrow i_{qr}^e = -\frac{1}{r_r}(\omega_e - \omega_r)\lambda_{dr}^e - p\lambda_{qr}^e$$

$$0 = r_r i_{dr}^e - (\omega_e - \omega_r)\lambda_{qr}^e + p\lambda_{dr}^e \Rightarrow i'^e_{dr} = \frac{1}{r_r}(\omega_e - \omega_r)\lambda_{qr}^e - p\lambda_{dr}^e \qquad (2)$$

$$0 = r_r i_{or}^e + p\lambda_{or}^e \Rightarrow i_{or}^e = -\frac{p\lambda_{or}^e}{r_r}$$

Since steady state operation of IM is considered, the time derivative term of flux linkage in **equation 2** will vanish. Hence, the rotor currents are:

$$i_{qr}^{e} = -\frac{1}{r_r}\left(\omega_e - \omega_r\right)\lambda_{dr}^{e}$$

$$i_{dr}^{e} = \frac{1}{r_r}\left(\omega_e - \omega_r\right)\lambda_{qr}^{e} \qquad (3)$$

$$i_{or}^{e} = 0$$

The dot product of the rotor flux linkage and rotor current vectors may be expressed as

$$\lambda_{qdr}^{e} \cdot i_{qdr}^{e} = \lambda_{qr}^{e} \cdot i_{qr}^{e} + \lambda_{dr}^{e} \cdot i_{dr}^{e} \qquad (4)$$

For

m **equation 5** it can be seen that the dot product between the rotor flux and rotor current vectors is zero in case of singly excited IM. Hence, it can be concluded that the rotor flux and rotor current vectors are perpendicular to each other in steady state operation. The defining feature of FOC is that this characteristic (that the rotor flux and rotor current vectors are perpendicular to each other) is maintained during transient conditions as well.

(7)

By suitable choice of θ_s on an instantaneous basis, **equation 6** can be achieved. Satisfying **equation 7** can be accomplished by forcing d -axis stator current to remain constant. To see this, consider the d -axis rotor voltage equation

$$0 = r_r\, i_{dr}{}^e + (\omega_e - \omega_r)\lambda_{qr}{}^e + p\lambda_{dr}{}^e$$

Since $\lambda_{qr}{}^e = 0$, **equation 8** can be written as

$$0 = r_r\, i_{dr}{}^e + p\lambda_{dr}{}^e$$

Substituting the values of i_{qr}^e and i_{dr}^e from **equation 3** into **equation 4** gives

$$\lambda_{qdr}{}^e \, . i_{qdr}{}^e = -\frac{\lambda_{qr}^e}{r_r}(\omega_e - \omega_r)\lambda_{dr}{}^e + \frac{\lambda_{dr}^e}{r_r}(\omega_e - \omega_r)\lambda_{qr}{}^e = 0$$

(6)

In both direct and indirect FOC, the 90^o vector can be achieved in two steps:

- The first step is to ensure that

$$\lambda_{qr}{}^e = 0$$

- The second s

The d -axis rotor flux link

age is given by (refer Lecture 19):

$$\lambda_{dr}^e = L_{lr}\, i_{dr}^e + L_m \left(i_{ds}^e + i_{dR}^e\right)$$

Substituting the value of λ_{dr}^e from **equation 10** into **equation 9** gives: (11)

$$pi_{dr}^e = -\frac{r_r}{L_{lr}} i_{dr}^e - \frac{L_m}{L_{lr}} pi_{ds}^e$$

If i_{ds}^s is held constant, then $pi_{ds}^e = 0$ and the solution of **equation 11** becomes (12)

$$i_{dr}^e = Ce^{-\left(\frac{r_r'}{L_{lr}}\right)t}$$

where

□ is a constant of integration

.

It is evident from **equation 12** that the rotor current i_{dr}^e will decay to zero and stay at zero regardless of other transients that may be taking place. Hence, the torque (from **equation 2**) is given by

$$T_e = \frac{3}{2}\frac{P}{2}\lambda_{dr}^e i_{qr}^e \qquad (13)$$

The q -axis rotor flux is given by (refer Lecture 19)

$$\lambda_{qr}^e = L_{lr} i_{qr}^e + L_m\left(i_{qs}^e + i_{qr}^e\right) \qquad (14)$$

Since, $\lambda_{qr}^e = 0$, the q -axis rotor current is given by

$$i_{qr}^e = -\frac{L_m}{L_{lr}} i_{qs}^e \qquad (15)$$

Combining **equations 13** and **15** gives

$$T_e = \frac{3}{2}\frac{P}{2}\frac{L_m}{L_{lr}}\lambda_{dr}^e i_{qs}^e \qquad (16)$$

The d -axis rotor flux is given by (refer Lecture 19)

$$\lambda_{dr}^e = L_{lr} i_{dr}^e + L_m\left(i_{ds}^e + i_{dr}^e\right) \qquad (17)$$

The **equation 7** gives $i_{dr}^e = 0$, hence **equation 17** can be written as

$$\lambda_{dr}^e = L_m i_{ds}^e \qquad (18)$$

Together, **equation 19** and **equation 21** suggest the generic rotor flux oriented control shown in **Figure 2**.

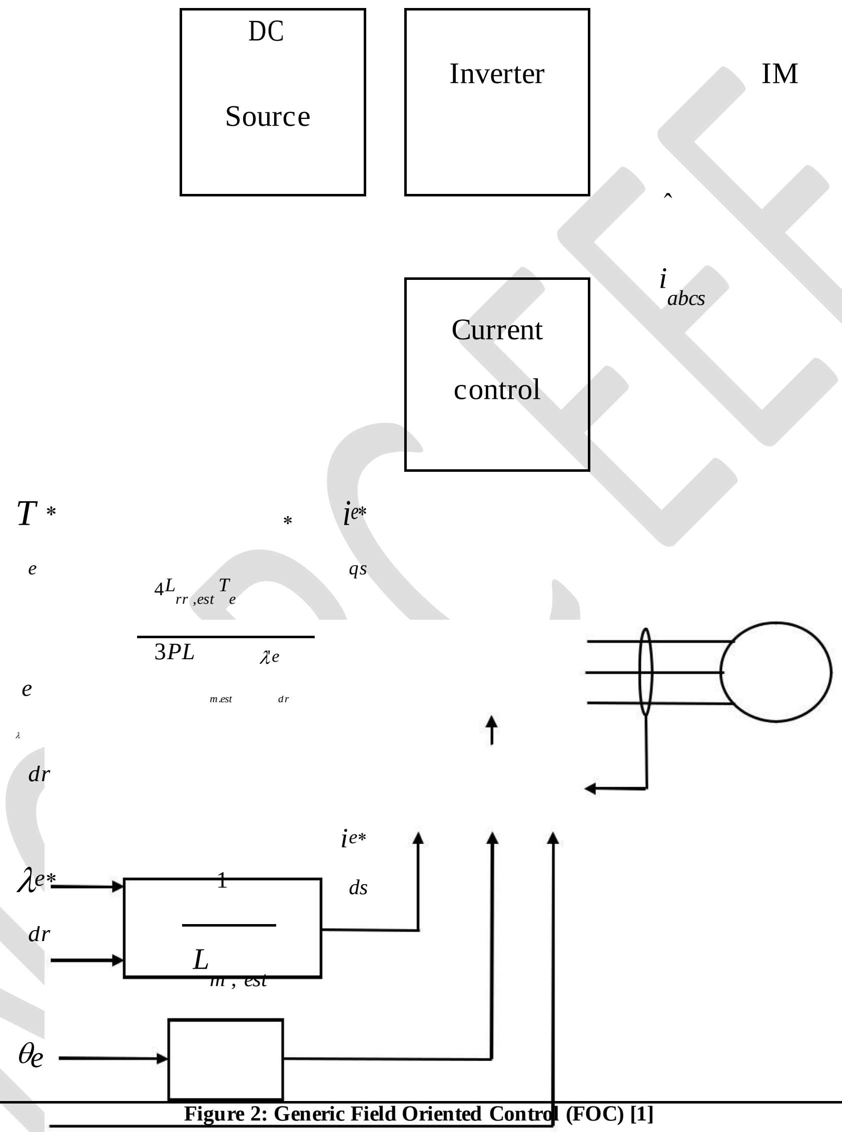


Figure 2: Generic Field Oriented Control (FOC) [1]

In **Figure 2** the variables of the form x^*, x and $\hat{x}$ denote ***command***, ***measured*** and ***estimated*** values respectively. In case of ***parameters that are estimated***, a subscript "**est**" is used. The working of the controller is as follows:

- Based on the torque command (T_e^*), the assumed values of the parameters and the estimated value of d -axis rotor flux $\hat{\lambda}_{dr^s}$ is used to formulate a q -axis stator current command i_{qs}^{s*} .

- The d -axis stator current command $i_{ds^s}^{*}$ is calculated such as to achieve a rotor flux command λ_{dr}^{s*} (using **equation 12**).s
- The q -axis and d -axis stator current command is then achieved using a current source control.

The above description of rotor flux oriented FOC is incomplete with determination of $\hat{\lambda}_{dr^s}$ and θ_s . The difference between ***direct*** and ***indirect*** FOC is in how these two variables are determined.

Direct Rotor Oriented FOC

In direct FOC, the position of the synchronous reference frame (θ_e) is determined based on the values of q -axis and d -axis rotor flux linkages in the stationary reference frame. The relation of flux linkages in synchronous reference frame and stationary reference frame is

$$\begin{bmatrix} \lambda_{qr}^{e} \\ \lambda_{dr}^{e} \end{bmatrix} = \begin{bmatrix} \cos\theta_e & -\sin\theta_e \\ \sin\theta_e & \cos\theta_e \end{bmatrix} \begin{bmatrix} \lambda_{qr}^{s} \\ \lambda_{dr}^{s} \end{bmatrix} \tag{19}$$

where

λ_{qr}^{s} is the rotor q-axis flux linkage in stationary frame of reference

λ_{dr}^{s} is the rotor d -axis flux linkage in stationary frame of reference

In order to achieve $\lambda_{qre} = 0$, it is sufficient to define the position of the synchronous reference frame as

$$\theta_e = \tan^{-1}\left(\frac{\lambda_{qr}^{s}}{\lambda_{dr}^{s}}\right) + \frac{\pi}{2} \tag{20}$$

The difficulty with this approach is that λ_{qrs} and λ_{drs} are not directly measurable quantities.

However, they can be estimated using direct measurement of air gap flux. To measure the air gap flux, hall-effect sensors are placed in the air gap and used to measure the air-gap flux in q -axis and d -axis. Since the hall-effect sensors are stationary, the flux measured by them is in stationary reference frame. The flux measured by the sensors is the net flux in the air gap (combination of stator and rotor flux). The net flux in the air gap is given by:

$$\lambda_{qm}^{s} = L_m\left(i_{qs}^{s} + i_{qr}^{s}\right) \tag{21}$$

where

L_m is the magnetization inductance

From **equation 21**, the rotor q-axis current is obtained as

$$i_{qr}^s = \frac{\lambda_{qm}^s - L_m i_{qs}^s}{L_m} \tag{22}$$

The q-axis rotor flux linkage is given by

$$\lambda_{qr}^s = L_{lr}\, i_{qr}^s + L_m\left(i_{qs}^s + i_{qr}^s\right) \tag{23}$$

Substituting the rotor q-axis current from **equation 22** into **equation 23** gives

$$\lambda_{qr}^s = \frac{L_{lr}}{L_m}\lambda_{qm}^s - L_{lr}\, i_{qs}^s \tag{24}$$

.

An identical derivation for d -axis gives

$$\lambda_{dr}^{s} = \frac{L_{r}}{L_{m}} \lambda_{dm}^{s} - L_{lr} i_{ds}^{s} \quad (25)$$

The implementation of this control strategy is shown in **Figure 3a** and **b**.

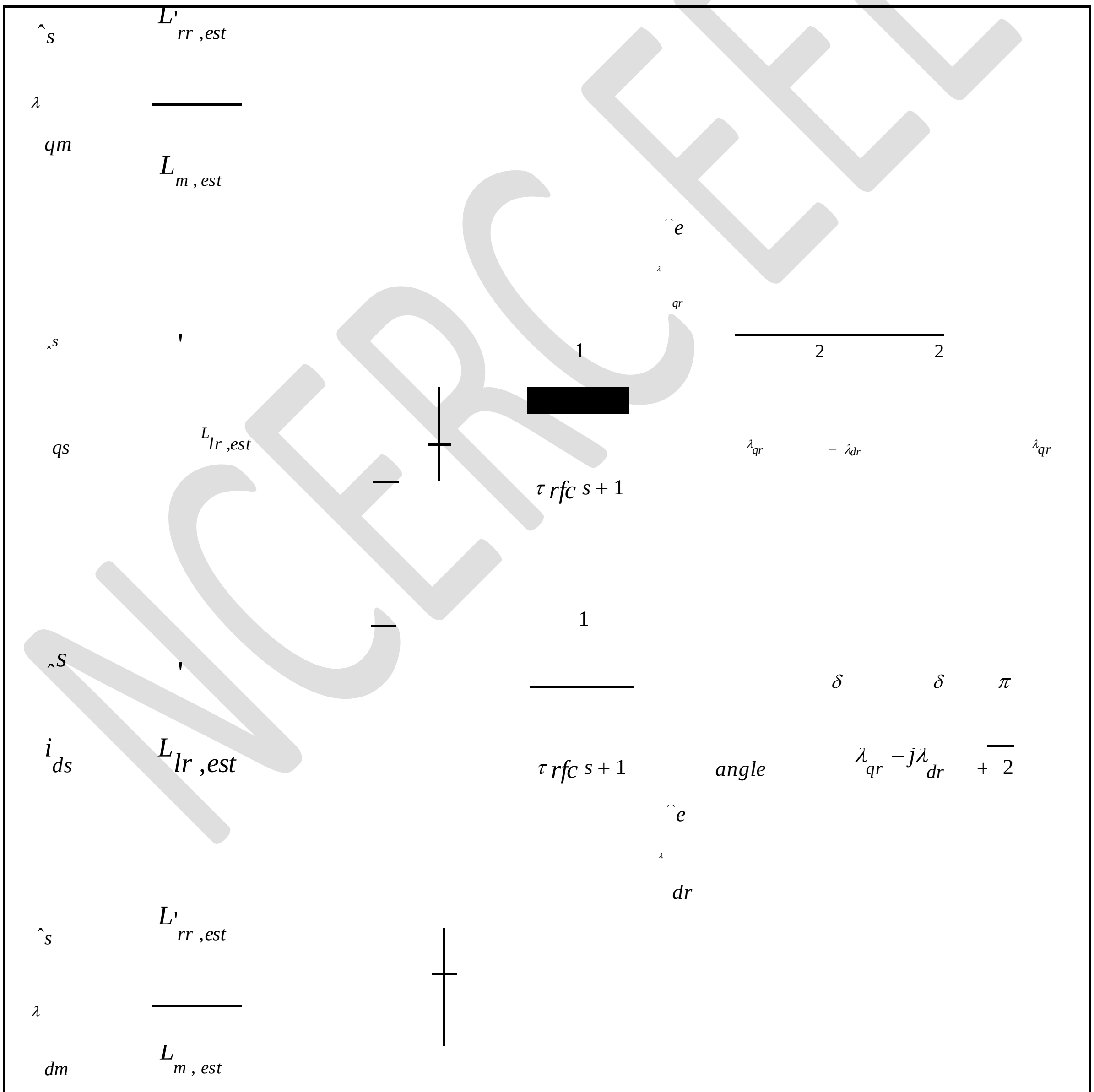


Figure 3a: Rotor flux calculation in Direct FOC [1]

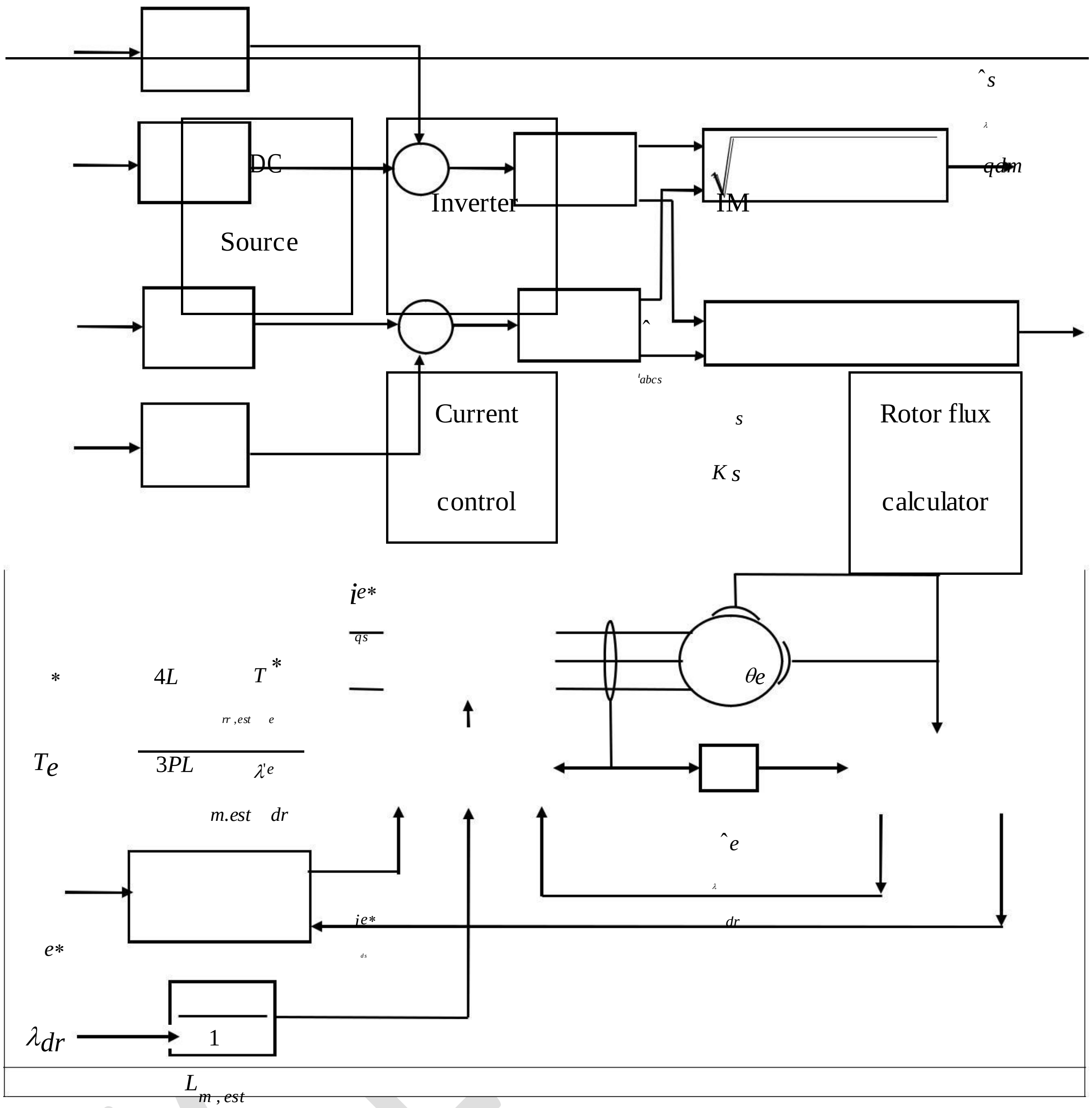


Figure 3b: Control strategy for Direct FOC [1]

Indirect Rotor Oriented FOC

The direct FOC is problematic and expensive due to use of hall-effect sensors. Hence, indirect FOC methods are gaining considerable interest. The indirect FOC methods are more sensitive to knowledge of the machine parameters but do not require direct sensing of the rotor flux linkages.

The q -axis rotor voltage equation in synchronous frame is

$$0 = r_r\, i_{qr}^{e} + (\omega_e - \omega_r)\lambda_{dr}^{e} + p\lambda_{qr}^{e} \qquad (26)$$

Since $\lambda_{qr}^{e} = 0$ for direct field oriented control, **equation 26** becomes

$$0 = r_r\, i_{qr}^{e} + (\omega_e - \omega_r)\lambda_{dr}^{e}$$

$$\Rightarrow \omega_e = \omega_r - r_r \frac{i_{qr}^{e}}{\lambda_{dr}^{e}} \qquad (27)$$

Substituting the values of i_{qre} and λ_{dr}^{e} from **equation 15** and **18** respectively into **equation**

□ gives

$$\omega_e = \omega_r + \frac{r_r}{L_{lr}} \frac{i_{qs}^{e}}{\lambda_{ds}^{e}} \qquad (28)$$

From **equation 28** it can be observed that instead of establishing θ_e using the rotor flux as shown in **Figure 3**, it can be determined by integrating ω_e given by **equation 28** where ω_e is given as:

$$\omega = \omega + \frac{r_r}{L} \frac{i_{qs}^{e*}}{\lambda^{e*}} \qquad (29)$$

er

lr ds

The **equation 29** does satisfy the conditions of FOC. In order to check it, consider the rotor voltage equations for the q -axis and d -axis:

$$0 = r_r\, i_{qr}{}^{e} + (\omega_e - \omega_r)\lambda_{dr}{}^{e} + p\lambda_{qr}{}^{e} \tag{30}$$

$$0 = r_r\, i_{dr}{}^{e} + (\omega_e - \omega_r)\lambda_{qr}{}^{e} + p\lambda_{dr}{}^{e} \tag{31}$$

Substituting ω_e from **equation 29** into **equations 30** and **31** gives

$$0 = r_r\, i_{qr}^{e} + \frac{r_r'\, i_{qs}^{e*}}{L_{lr}'\, i_{ds}^{e*}}\lambda_{dr}^{e} + p\lambda_{qr}^{e} \tag{32}$$

$$0 = r_r\, i_{dr}^{e} + \frac{r_r'\, i_{qs}^{e*}}{L_{lr}'\, i_{ds}^{e*}}\lambda_{qr}^{e} + p\lambda_{dr}^{e} \tag{33}$$

Substituting the value of d-axis rotor flux from **equations 17** into **equation 33** gives

$$0 = r_r \left(\frac{\lambda_{qr}^e - L_m i_{qs}^{e*}}{L_{lr}} \right) i_{qr}^e + \frac{r_r i_{qs}^{e*}}{L_{lr} i_{ds}^{e*}} \left(L_{lr} i_{dr}^e + L_m i_{ds}^{e*} \right) + p\lambda_{qr}^e \quad (34)$$

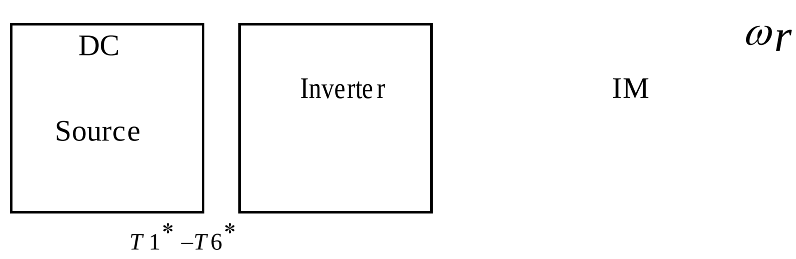
$$0 = r_r i_{dr}^e - \frac{r_r i_{qs}^{e*}}{L_{lr} i_{ds}^{e*}} \lambda_{qr}^e + p\left(L_{lr} i_{qr}^e + L_m i_{ds}^{e*} \right) \quad (35)$$

If the d-axis rotor current is held constant, then $pi_{dr}^{e*} = 0$ and rearranging **equations 34** and **35** gives

$$p\lambda_{qr}^e = -\frac{r_r}{L_{lr}} \lambda_{qr}^e - r_r \frac{i_{qs}^{e*}}{i_{ds}^{e*}} i_{dr}^e \quad (36)$$

$$pi_{dr}^e = -\frac{r_r}{L_{lr}} \lambda_{dr}^e + \frac{r_r i_{qs}^{e*}}{L_{lr} i_{ds}^{e*}} \lambda_{dr}^e \quad (37)$$

The solution of **equations 37** and **38** will decay to zero (same argument as used for **equation 12**), hence λ_{qr}^e and i_{qr}^e will eventually become zero. In **Figure 4** the implementation of ***indirect FOC*** is shown and it is much simpler than the ***direct FOC***.



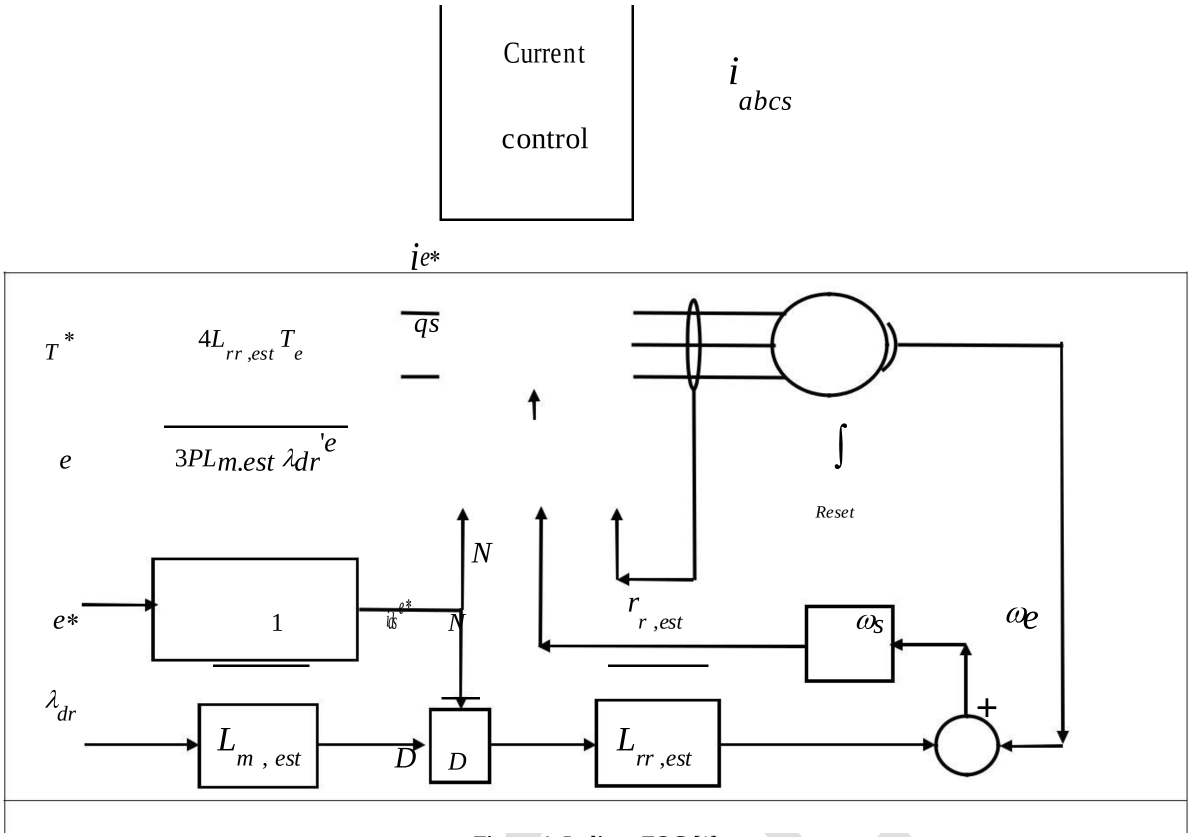


Figure 4: Indirect FOC [1]

References:

□ P. C. Krause, O. Wasynczuk, S. D. Sudhoff, *Analysis of electric machinery*, IEEE Press, 1995

Suggested Reading

□ R. Krishnan, *Electric motor drives: modeling, analysis, and control*, Prentice Hall, 2001

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